

proof of the VI. and of the XVII. propositions.

2. To make a triangle of which the sides shall be equal to three straight lines, but any two of these must be greater than the third.

3. If a straight line fall upon two parallel straight lines it makes the two interior angles upon the same side together equal to two right angles, and also the alternate angles equal to each other, and also the exterior angle equal to the interior and opposite angle upon the same side.

4. In any right-angled triangle the square which is described on the side subtending the right angle is equal to the squares described on the sides containing the right angle.

5. If a straight line be divided into any two parts the squares on the whole line and on one of the parts are equal to twice the rectangle contained by the whole and that part together with the square on the other part.

6. To describe a square that shall be equal to a given rectilinear figure.

7. In the ordinary figure of the 47th proposition, Book I., if the corners of the squares be joined externally, prove that the three triangles thus formed are equal to one another.

8. If ABCD be a quadrilateral, and E the bisection of the diagonal BD, and if through E a line, FEG, be drawn parallel to AC, and meeting AB in F and BC in G, shew that AG will bisect the given figure.

9. If A be the vertex of an isosceles triangle ABC, and CD be drawn perpendicular to AB, prove that the squares upon the three sides are together equal to the square on BD, and twice the square on AD and thrice the square on CD.

10. Any rectangle is half the rectangle contained by the diameters of the squares upon its two sides.

ALGEBRA.

1. Simplify.

$$\left(\frac{ax^2 - ay^2 + 2bxy}{x^2 + y^2} \right)^2 + \left(\frac{by^2 - bx^2 + 2axy}{x^2 + y^2} \right)^2$$

2. Divide $a^3 - b^3 - c^3 - 3abc$ by $a - b - c$, and show, without expansion, that

$$(1 + x + x^2)^3 - (1 - x + x^2)^3 - 6x(x^4 + x^2 + 1) - 8x^3 = 0.$$

3. Resolve into factors $x^4 - \frac{1}{4}x^2y^2 + y^4$, and

$7x^2 - 6y^2 - xy + 19x + 33y - 36$; and prove that

$b^2(c+a) + c^2(a+b) - a^2(b+c) + abc$ is exactly divisible by $b + c - a$.

4. Apply Horner's method of division to find the value of $5x^5 + 497x^4 + 200x^3 + 196x^2 - 218x - 2000$ when $x = -99$, and the value of $6x^5 + 5x^4 - 17x^3 - 6x^2 + 10x - 2$ when $2x^2 = -3x + 1$.

5. Find what

$$\frac{\sqrt{(a+x)} + \sqrt{(a-x)}}{\sqrt{(a+x)} - \sqrt{(a-x)}} \text{ becomes when } x = \frac{2ab}{1+b^2}.$$

6. If a and b be any positive numbers, prove that

$$\frac{1}{a} + \frac{a}{1+a} > 1, \quad \frac{a}{b} + \frac{b}{a} > 2.$$

7. Solve the equations—

$$(1) \quad x^{\frac{1}{2}} + y^{\frac{1}{2}} = 5$$

$$\frac{-\frac{1}{2}}{x} + \frac{-\frac{1}{2}}{y} = \frac{1}{8}.$$

$$(2) \quad x + 2y + 3z = 14$$

$$2x + 3y + z = 11$$

$$3x + y + 2z = 11.$$

$$(3) \quad (x+1)(x+3)(x+4)(x+6) = 16.$$

8. There are three consecutive numbers such that the sum of their cubes is equal to $16\frac{2}{3}$ times the product of the two higher numbers: find the numbers.

9. (1) Form an equation three of whose roots are 0, $\sqrt{-3}$, and $1 - \sqrt{2}$.

(2) If one of the roots of the equation $x^2 + px + q = 0$, is a mean proportional between p and q , prove that $p^3 = q(1 + p)^2$.

10. Two trains start at the same instant, the one from B to A, the other from A to B; they meet in $1\frac{1}{2}$ hours; and the train for A reaches its destination $52\frac{1}{2}$ minutes before the other train reaches B: compare the rates of the trains.

NATURAL PHILOSOPHY.

1. How are static forces measured?

State the principle of the transmissibility of force.