4. (*a*) Prove that $\sin \alpha + \sin \beta = 2 \sin \frac{1}{2} (\alpha+\beta) \cos \frac{1}{2} (\alpha-\beta)$, and write the corresponding value for $\cos \alpha + \cos \beta$.

(*b*) Show that $\sin 41^\circ + \sin 67^\circ - \sin 31^\circ - \sin 77^\circ = \sin 5^\circ$, given $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

(*c*) Express $\cos m\theta . \cos n\theta . \cos p\theta$ as the sum of four cosines.

4. (*a*) Book work.

(*b*) $\sin 41^\circ - \sin 31^\circ + \sin 67^\circ - \sin 77 = 2 \cos 36^\circ \sin 5^\circ - 2 \cos 72^\circ \sin 5^\circ = 2 \sin 5^\circ (\cos 36^\circ - \cos 72^\circ) = 4 \sin 5^\circ (\sin 54^\circ \sin 18^\circ) = 4 \sin 5^\circ \sin 18^\circ (\cos 36^\circ) = 4 \sin 5^\circ \sin 18^\circ (1 - 2\sin^2 18^\circ). = \sin 5^\circ, 4 \sin 18^\circ (1 - 2 \sin^2 18^\circ) = 1.$

(*c*) $2 \cos A \cos B = \cos (A+B) + \cos (A-B). \therefore \cos p\theta, \cos m\theta, \cos n\theta = \cos p\theta (\cos m\theta, \cos n\theta),$

$= \frac{1}{2} \cos p\theta [\cos (m+n)\theta + \cos (m-n)\theta]$

$= \frac{\cos p\theta . \cos(m+n)\theta + \cos p\theta . \cos(m-n)\theta}{2}$

$= \frac{\cos (m+n+p)\theta + \cos (m+n-p)\theta + \cos (m-n+p)\theta + \cos (m-n-p)\theta}{4}$

5. In any triangle prove that :

(*a*) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}$.

(*b*) $\tan \frac{1}{2} (B - C) = \frac{b-c}{b+c} \cot \frac{1}{2} A.$

(*c*) $\tan A + \tan B + \tan C = \tan A . \tan B . \tan C.$

5. (*a*) Book work. (*b*) Book work. (*c*) Book work.

6. Find an expression for

(*a*) The radius of the incircle (inscribed circle) of a triangle.

(*b*) The radius of an excircle (escribed circle) of a triangle.

(*c*) The radius of the circumcircle (circumscribing circle) of a triangle.

6. (*a*) Book work. (*b*) Book work. (*c*) Book work.

7. (*a*) Prove that

$r_1 \cot \frac{A}{2} = r_2 \cot \frac{B}{2} = r_3 \cot \frac{C}{2} = r \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}.$

(*b*) The centres of the excircles of a triangle are joined. Show that the area of the triangle so formed is $\frac{abc}{2r}$.

7. (*a*) $r_3 \cot \frac{C}{2} = r \cot \frac{A}{2}, \cot \frac{B}{2} \cot \frac{C}{2}$

if $\frac{\Delta}{s-c} = \frac{\Delta}{s} \sqrt{\frac{s.s-a}{s-b. \ s-c}} \sqrt{\frac{s.s-b}{s-a, \ s-c}}$

(i.e.) $\frac{1}{s-c} = \frac{1}{s-c}$.

Similiarly for $r_1 \cot \frac{A}{2}$ and $r_2 \cot \frac{B}{2}$.

(*b*) Area of escribed triangle

$= \Delta + \frac{a}{2} r_1 + \frac{b}{2} r_2 + \frac{c}{2} r_3$

$= \Delta + \frac{a}{2} . \frac{\Delta}{s-a} + \frac{b}{2} . \frac{\Delta}{s-b} + \frac{c}{2} \frac{\Delta}{s-c}$

$= \Delta \frac{2s-a.s-b.s-c. + a(s-b)(s-c) + b(s-a)(s-c) + c(s-a)(s-b)}{2.(s-a)(s-b)(s-c)}$

$= \Delta . 2 \frac{\{s^3 - 2s^3 + s(ab+bc+ca) - abc\} + 2s^3 + 3abc - 2s(ab+bc+ca)}{2.s-a.s-b.s-c}$

$= \frac{s\Delta}{2\Delta^2} (abc) = \frac{s}{2\Delta} (abc) . = \frac{abc.s}{2\Delta}$

now $\frac{abc}{2r} = \frac{abc.s}{2\Delta}$.

EUCLID.

NOTE.—Candidates must take all the questions of section A, and any four of section B.

A.

1. (*a*) Upon the same base, and on the same side of it there cannot be two triangles that have their sides which are terminated in one extremity of the base, equal to one another, and likewise those which are terminated in the other extremity.—Euc. I, 7.

(*b*) Prove the foregoing directly from the assumed axiom, "a straight line is the shortest distance between two given points."

2. (*a*) All the interior angles of any rectilineal figure together with four right angles, are equal to twice as many right angles as the figure has sides. Euc. I, 32, Cor. 1.