$$\begin{aligned} f(x, y, z) = {} & f(1, 1, 1)\, xyz + f(1, 1, 0)\, xy\,(1 - z) \\ & + f(1, 0, 1)\, xz\,(1 - y) + f(1, 0, 0)\, x\,(1 - y)\,(1 - z) \\ & + f(0, 1, 1)\, yz\,(1 - x) + f(0, 1, 0)\, y\,(1 - x)\,(1 - z) \\ & + f(0, 0, 1)\, z\,(1 - x)\,(1 - y) \\ & + f(0, 0, 0)\,(1 - x)\,(1 - y)\,(1 - z) \quad \ldots\ldots\ldots (12) \end{aligned}$$

As the object of the expansion of logical symbols may not be evident at first sight, and as the process may consequently be regarded by some as barbarous, we may observe that not only is there a definite aim in the development, but the thing aimed at, has, in our opinion, been most felicitously accomplished. Of this our readers will probably be satisfied when they are introduced to some specimens of the use which is made of the formulæ obtained; in the meantime it may throw some light on the character of these formulæ if we notice that the constituents of an expansion represent the several exclusive divisions of what our author terms the universe of discourse, formed by the predication and denial in every possible way of the qualities denoted by the literal symbols. In the simplest case, that in which the function is one of a single concept, it will be seen by a glance at (10) that there are only two such possible ways, x and $1 - x$. In the case of a function of two symbols, there are [see (11)] four such ways, xy, $x\,(1 - y)$, $y\,(1 - x)$, $(1 - x)\,(1 - y)$. In a function of three symbols there are eight such ways; and so on. A development in which the constituents are of this kind prepares the way for ascertaining all the possible conclusions, in the way either of affirmation or denial, that can be deduced, regarding any concept, from any given relations between it and the other concepts.

If S be the sum of the constituents of an expansion, and P the product of any two of them, then

$$S = 1, \quad \ldots\ldots\ldots (13)$$

$$\text{and } P = 0. \quad \ldots\ldots\ldots (14)$$

The truth of these beautiful and important propositions will easily be gathered by an intelligent reader from an inspection of the formulae, (10), (11), (12). Another important proposition is involved in (14), namely, that, if $f(x) = 0$, either the constituent or the coefficient in every term of the expansion of $f(x)$ must be zero. For, let

$$f(x) = Q + A\,X + A_1\,X_1 + \ldots\ldots + A_n\,X_n;$$

where A, A_1, &c., are the coefficients which are not zero, their corresponding constituents being X, X_1, &c.; while Q represents the sum