curve; if stopped at 2, it is a transition curve to a 2° curve, etc.; therefore if the transition *does* continue past the points 1, 2, 3, etc., we may consider it to be composed of two parts: 1st, a 1°, or 2°, or 3°, etc., curve, according to circumstances. 2nd. Plus the *foundation series* of 3′, 12′, 27′, 48′, etc., beginning at the point considered, and continuing forward to any desired extent, and the transition curve deflections are the sum of these two. Also, in the same way, the transition curve deflections looking backward, with transit at any point, are those of a certain degree of curve corresponding to this point, minus the same *foundation series*; (e.g.) suppose the transit to be at the point 3, with the vernier at zero, and line of sight tangent to the curve, then the vernier readings to each intermediate point would be—

$$(A) - \left(\frac{90}{100} \times \frac{180'}{2} - 27'\right) = -0^\circ\ 54'.$$
$$(1) - \left(\frac{60}{100} \times \frac{180'}{2} - 12'\right) = -0^\circ\ 42'.$$
$$(2) - \left(\frac{30}{100} \times \frac{180'}{2} - 3'\right) = -0^\circ\ 24'.$$
$$(3)\quad 0^\circ\ 0' = 0^\circ\ 0 \text{ position of transit.}$$
$$(4) + \left(\frac{30'}{100} \times \frac{180'}{2} + 3'\right) = +0^\circ\ 30'.$$
$$(5) + \left(\frac{60}{100} \times \frac{180'}{2} + 12'\right) = +1^\circ\ 06'.$$
$$(6) + \left(\frac{90}{100} \times \frac{180'}{2} + 27'\right) = +1^\circ\ 48'.$$
$$(7) + \left(\frac{120}{100} \times \frac{180'}{2} + 48'\right) = +2^\circ\ 36'. \text{ etc.}$$

In this way a table can be prepared giving deflections to be made to any point (every 30 feet), with transit located at any point. These tables are conveniently made out by Mr. Armstrong, for 30-foot chords = 1 rail length; but different foundation series and different tables may be made out, or special calculations made by equations (12) to (20) for a transition curve of any rapidity of sharpening, but of the same nature and handled in the same way. This is often necessary where there is not room between the *BC* of one curve and *EC* of the previous one to permit of the introduction of transitions which sharpen so slowly as 30 feet per degree.

In street railway work, for instance, transitions sharpening from 0° to 20°, or even 40°, etc., are needed, and must not occupy more than 20 or 30 feet in length. Special corrections must be applied in such a case, and even for steam railways Mr. Armstrong has worked out corrections in lengths to apply to the very approximate equations here given, but as the correction is zero until an 8° curve is reached, and only 1 foot in 300 for a 10° curve, it is hardly worth taking account of here. Any one desiring extreme accuracy for curves from 8° upward, are referred to J. S. Armstrong's pamphlet.

The three problems most frequently met with in practice are briefly as follows:

1. (See Fig. 16.) To keep tangents fixed and to move the circular curve inward, retaining the same degree of curvature. In this case, take an arbitrary offset or length of transition, and determine the other unknowns by foregoing equations. The distance from the apex of tangents to the *B T C* consists of three parts:

(*a*) Sub-tangent of circular curve $= R \times \tan\frac{\theta}{2}$ (R = radius).

(*b*) Correction of shift $= O \times \tan\frac{\theta}{2}$. (See Fig. 16).

(*c*) ½ length of transition = n.

The amount in (*b*) is usually very small, unless θ is large.

2. (See Fig. 17.) To keep the circular curve fixed, and move out the tangents either in direction or position, or both: If the tangents are moved outward and kept parallel to their original positions, proceed as in (1), except that the correction of shift (*b*) does not exist. If the tangents are not moved outward parallel to their original positions, but pivoted about some distant point, then calculate the angle pivoted, and continue the circular curve through an equal central angle. So that a tangent to the curve at the new *B C* or *E C* would be parallel to the pivoted tangent; then measure the amount of shift O, and by the ordinary equations calculate the unknowns; the amount of shift *O* could be calculated without any field work. No correction of shift is here necessary; this second case is most usually met with in revising location, and is very convenient often in the final slight movement of tangents or curves, by avoiding the running over again of the whole circular curves, often situated on a rough hillside or heavy bush, and yet enabling a tangent to be moved on to better ground.

3. (See Fig. 18). To sharpen a curve and introduce transitions, so that the track will not be altered in length; this problem is the one met with in re-running old track centres where transition curves have not been previously used.

The method of solution is to assume an external secant slightly less than the original one, by an amount = expected shift, *O*, + an arbitrary amount of five inches to ten inches, depending on the sharpness and total central angle of the circular curve; then calculate the transitions and complete position of a curve of assumed external secant and given total central angle, and, either by plotting or calculations, determine whether this new curve will cross the original one about at the ¼ points and give the same length of track, thereby minimizing the movement of the track. If in error, a second trial will give usually satisfactory results. This method will often be found to give transitions, which, unless the central angle is large, will occupy the whole central angle, leaving no circular curve at the centre. As this is not desirable, it is preferable in a case of this kind to use shorter and sharper transitions, so as to retain a considerable portion of circular curve at the centre.

While these are the three usual problems to solve, others may arise such as introducing a transition at a point of compound curvature which needs special solutions. For further details, the reader is referred to the literature already mentioned, and the engineer, young or old, who has not used transitions in the field, is advised to become familiar with some one of the forms given, and actually put it into practice, when its seeming tediousness and difficult nature will disappear.

He should recognize that, as he would be quite ready to spend a few hours extra now and then, during railway construction or maintenance, on trivial matters such as affect the general appearance of the road only, and are not really important, he should be far more willing to give much additional labor and attention to such a question as this, when the returns will be increased comfort to travellers, decreased wear on rolling stock, and greater ease in retaining good alignment at the ends of curves. Whenever transitions have been used, their beneficial effects have at once been recognized, and once established trackmen maintain them easily and instinctively. Some of the oldest and most conservative of the American roads are now engaged in introducing them on their permanent tracks.