L C d is equal to the whole angle L B a. In the triangles d C L and a B L, a B and B L are equal to d C and C L, and their contained angles equal, by (4.1), a L is equal to d L, and the angle B a L equal to the angle C d L, by (13.1), the angle r a b is equal to the angle l d c. In the triangle $r \perp O$ and $l \perp O$, the three sides of the one are respectively equal to the three sides of the other, by (8.1), O r L is equal to the angle O l L, and the angle r O L equal to the angle l O L; by (15.1) the angle arb and dlc are equal but it has been proved that the angle b a r is equal to c d l; therefore (32.1), the two triangles are equiangular, and a r equal to dl; therefore, by (26.1), ab is equal to cd, but aBis equal dC; therefore b B is equal to cC. Then, in the triangles b B O and c C O, we have b B and B Oequal to c C and C O and their contained angles equal, by (4.1), the angle BO b is equal to the angle CO c. Then, as in the first demonstration it may be proved that each of these angles, is equal to the angle $b \circ c$; therefore the three angles B O r, r O l, and l O C, are equal, and the angle B O C is trisected.

THIRD DEMONSTRATION.

Join B l and C r, cutting O l and O r in n and m. If on O B we describe a semicircle, it passes through the point m; therefore (31.3), the angles at m are right angles, and (3.3) B l is bisected perpendicularly by O m; hence B m and m O are equal to O m and m l and the angles at m equal, by (4.1), the angle B O m is equal to the angle l O m. This may also be proved by letting fall a perpendicular from O, on B l and this perpendicular always falls on the point m; then we have two straight lines which coinside in part they must coinside throughout the whole (Legendre, Prop. 2). It

1.1