contains a minimum amount of material when the central angle is kept 133° at all elevations.

The curve in Fig. 2 shows this graphically. The abcissas represent the central angle 2θ and the ordinates represent the term $\frac{\theta}{\sin^2\theta}$, the latter being proportional to the volume of masonry. In addition to showing the point of maximum economy, this curve also shows that as long as the subtended angle is kept above 110° the variation in the amount of masonry is very small, but below 110° the volume increases rapidly. Most all dam sites are narrower at the bottom than they are at the crest elevation; therefore, in order to place the material in the dam most economically, it is necessary to change the length of the mean radius of the dam continuously from the bottom to the top corresponding to the change in width of the site, so as to keep the subtended central angle constant. In practice it is seldom possible to keep this angle exactly constant. It is a mere ideal condition, but one should try to bring practice as close to theory as possible in designing the arch.

To prevent upper portions of the dam from overhanging lower portions, it will be necessary to have the thickness of the section increase from the crest towards the foundation. The proportional increase in water pressure must therefore be greater than the proportional decrease in length of the upstream radius towards the foundation. The ratio of increase in water pressure is always fixed, and the ratio of decrease in the length of the upstream radius depends upon the slope of the canyon sides. If these slopes are such, that at any intermediate elevation the ratio of decrease in length of the upstream radius has been greater than the ratio of increase in water pressure, a decrease in thickness of the dam at this elevation would result, and the structure would be overhanging, which is impractical.

Whenever a certain thickness must be provided to prevent overhanging, it is most economical to increase the length of the mean radius above that corresponding to a central angle of 133° for the reason that a flat arch requires less material than a more curved one of the same thickness.

In the foregoing the thickness of different arch slices at different elevations have been determined, from (1), as if all the load were taken by the arch and the dam had no gravity action at all. How safe a dam would result depends primarily upon the unit compression allowed when using (1) for finding the thickness at different elevations. This design, however, would in most cases prove to be weakest in the middle, for the same reason that a long column held at both ends is weakest in the middle and on account of having highest cantilever stresses here. Whenever t is small compared with R_u, the arch, when loaded, is practically a long column in compression, and the length of the arch should therefore not be over 25 times its thickness if the material is to be highly stressed. It is true that this circular column is supported to some extent along one side, but this added stiffness may be largely offset by the fact that the water may not soak through the upstream face uniformly, i.e., the effect of the water pressure would in all probability be unsymmetrical about the centre line of the dam. On a high, comparatively thin arch dam section, the resulting compression due to cantilever action and weight of material above may become excessive near the foundation requiring some additional material along the downstream face towards the foundation. The thickness of this added material should decrease vertically from a maximum at the foundation to zero at some higher ele-

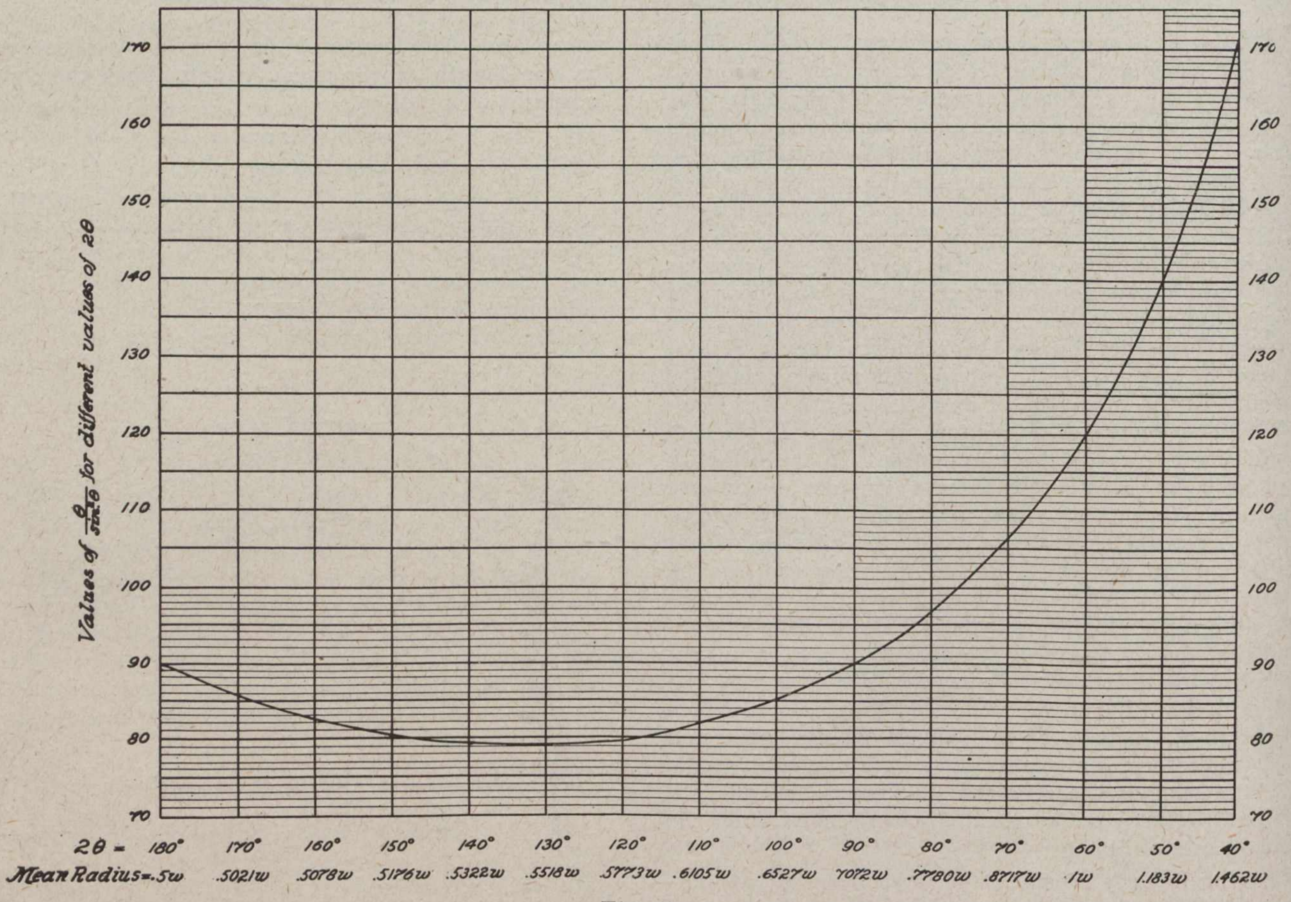

Fig. 2.