SOLUTIONS.

12. Let ABC be the triangle, and $A = B + C$. Make $CAD = ACD$; then $BAD = ABD$; $\therefore CD = DA = DB$.

13. Make angles at the ends of the given diagonal and on opposite sides of it equal to the middle point of this diagonal draw a line parallel to the line given in position.

14. An extension of Prop. 44, Bk. I.

15. A, B be the points, O the centre, CBD the chord. Then $AC^2 + CB^2 = 2CO^2 + 2OB^2 =$ const.; so $AD^2 + DB^2 =$ const.; and rectangle $CB.BD$ is const. Hence $AC^2 + AD^2 = CB^2 + BD^2 + 2CB\,BD$ is const.; $\therefore AC^2 + AD^2 + CD^2$ is const.

16. The triangles DCA, DAB are similar.

$$\therefore \frac{CD}{CA} = \frac{DA}{AB} \text{ and } \frac{AB}{BD} = \frac{AC}{AD}$$

$$\therefore \frac{CD}{BD} \cdot \frac{AB}{CA} = \frac{AC}{AB}, \text{ or } \frac{CD}{BD} = \frac{AC^2}{AB^2}.$$

ALGEBRA—HONORS.

Examiner: A. K. Blackadar, B.A.

1. Multiply $a^{-2} - 2 + a^2$ by $a^{-1} - 2 + a$, and divide the product by $a^{-3} - a^3 - 2a(a^{-2} - 1)$.

2. Divide $\dfrac{1+x^3}{(1-x)^3}\left(\dfrac{1}{1-x} - \dfrac{x}{1-x^2} + \dfrac{x^2}{1-x^3}\right)$

by $\dfrac{1-x+x^2}{(1-x)(1-x^2)(1-x^3)}$.

3. Resolve into factors:

(1) $2x^3 - 6x^2 - x + 3$.

(2) $2ab + (a+b)\{(a-b)^2 + 3(a-b)^2\} + 8a^2b^2$.

If $a+b+c = 2s$, shew that

$s(s-b)(s-c) + s(s-c)(s-a) + s(s-a)(s-b) - (s-a)(s-b)(s-c) = abc$.

4. Define the terms *Common Divisor* and *Common Multiple*, and prove that every common multiple of two algebraic expressions is a multiple of their least common multiple.

If $p^2+pq+q^2=0$, shew that x^2+px+p^2 and x^2+qx+q^2 have a common divisor $x+p+q$, and a common multiple x^3-p^3 or x^3-q^3.

5. Find the values of x and y from the equations

$$\begin{cases} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{cases}$$

by the method of (1) substitution, (2) comparison, (3) elimination by means of arbitrary multipliers.

Find the relation between the constants when the values of x and y are indeterminate.

6. Solve the equations:

(1) $\dfrac{a}{x-a} + \dfrac{b}{x-b} + \dfrac{c}{x-c} = \dfrac{abc}{(x-a)(x-b)(x-c)}$.

(2) $\sqrt{x^2+3x-10} + x = \sqrt{x+5} - 5$.

(3) $\left.\begin{array}{l} xz + yx = xy, \\ x^2(5z+y) = 5yz, \\ 10z - 6x^2 = 10x^2z. \end{array}\right\}$

7. Show how to find the product of two simple surds $\sqrt[m]{a}$ and $\sqrt[n]{b}$.

From the equation $x^4 - 10x^2 + 1 = 0$, find the values of x in the form of the sum or difference of two surds; and the values of $\dfrac{1}{x}$ correct to three decimal places.

8. Insert m arithmetic means between a and b.

If 1 be the $(m+1)$th term in the $A.S.$ of which the first term is $\dfrac{m}{n}$ and the last term is $\dfrac{n}{m}$, shew that the sum of the series is

$$\frac{(m+n+1)(m^2+n^2)}{2mn}$$

9. Sum the series

$$\frac{3}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{2}\sqrt{3} + \dots$$

to $2n$ terms, and to infinity.

10. Find the number of permutations of n things taken r at a time.

Find the number of permutations of the letters in the word *Toronto*, taken all together.

How many different numbers, each containing 3 figures, can be formed out of the 10 digits, in each number two figures at least being alike?

11. Find the greatest term in the expansion of $(x+a)^n$, n being a positive integer.

Write down the 7th term in the expansion of the square root of $(1-\sqrt{x})^3$.

Shew that

$$n^{-n} = \left\{\frac{1}{\lfloor 2} - \frac{2n-1}{\lfloor 3} + \frac{(2n-1)(3n-2)}{\lfloor 4} - \dots\right\}^{n-1}$$

SOLUTIONS.

1. $= \dfrac{(a^{-1}-a)^2(a^{-1}-2+a)}{(a^{-1}-a)(a^{-1}+a-2)} = a^{-1} - a$.

2. $(1+x+2x^2+x^3)\,\dfrac{1+x}{1-x}$.

3. (1) $(x-3)(x\sqrt{2}+1)(x\sqrt{2}-1)$.

(2) $= 2ab + 4(a^3+b^3) + 8a^2 + b^2 = 2(a+2b^2)(b+2a^2)$.

(3) $= s\{3s^2 - 2(a+b+c)s + (ab+bc+ca)\} - \{s^3 - (a+b+c)s^2 + (ab+bc+ca)s - abc\}$, which, on substituting $2s$ for $a+b+c$, reduces to abc.

4. On substituting $-(p+q)$ for x in x^2+px+p^2 and x^2+qx+q^2, we see that $x+p+q$ is a factor of both provided $p^2+pq+q^2=o$. The factor will be contained in x^2-px+p^2 $x-q$ times, and in x^2+qx+q^2 $x-p$ times; hence C. M. is x^3-p^3 or x^3-q^3.

5. The values of x and y are indeterminate when they assume the form $\dfrac{0}{0}$; which requires $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$,

there being in this case evidently but one independent equation.

6. (1) $x = \dfrac{ab+bc+ca \pm \sqrt{a^2b^2+b^2c^2+c^2a^2}}{a+b+c}$.

(2) Equation becomes $\sqrt{(x+5)(x-2)} + (x+5) = \sqrt{(x+5)}$, $\therefore x+5=0$ gives one root, $x=-5$, and others are obtained from $\sqrt{x-2} + \sqrt{x+5} = 1$, where $x = \dfrac{-b \pm \sqrt{-148}}{6}$.

(3) From (2) & (3) subtracting $x^2 = \dfrac{5z(y-1)}{y-3}$

Also from same dividing $\dfrac{5z+y}{10y+6} = \dfrac{y}{2}$, or $z = \dfrac{2y}{5(1-y)}$.

From first of these with (1) $\dfrac{y^2z^2}{(y-z)^2} = \dfrac{5z(y-1)}{y-3}$, and substituting the value of z in terms of y, and simplifying $3y^2-4y+1=0$, *i.e.*, $y = 1$ or $\frac{1}{3}$, and thence the values of z and x may be obtained.

7. $\sqrt[m]{a} \times \sqrt[n]{a} = \sqrt[mn]{a^n} \times \sqrt[mn]{a^m} = \sqrt[mn]{a^{m+n}}$.

$x^2 = 5 \pm 2\sqrt{6}$; $\therefore x = \pm(\sqrt{3} \pm \sqrt{2})$

$$\frac{1}{x} = \frac{1}{\sqrt{2}+\sqrt{3}} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \cdot 317+.$$

8. From formula $l = a + (n-1)d$, $1 = \dfrac{m}{n} + m.d$, $\therefore d = \dfrac{n-m}{mn}$.