and hence a common normal PC at their point of contact, and this normal will pass through the pitch point C. Thus Pe and Pf fulfill the necessary conditions for the shapes of gear teeth. Evidently the points of contact of these two curves lie along PC, since both curves are described simultaneously by a point which always remains on the circle PC. Since these curves are first in contact at P and then again at C, when P, C, e and f coincide, it is evident that during the motion from P to C the curve Pe slips on the curve Pf through the distance $Pf - Pe$. Below C the pencil at P would simply describe the same curves over again, only reversed.

To further extend these curves, we place a second pencil at P^1, which will draw the curves P^1g and P^1h in the same way as before, these curves having the same properties as Pe and Pf, the amount of slipping in this case being $P^1g - P^1h$, and the points of contact always lying on the circle CP^1.

Now join the two curves formed on eg, that is, join gP^1 to Pe, as shown at $Peg^1P_1^1$, and then the two curves on fch, as shown at $Pfh_1P_1^1$, and we have a pair of curves which will remain in contact from P to P_1, which always have a point of contact on the curve PCP^1, and which always have a common normal at their point of contact passing through C. The relative amount of slipping is $Pfh_1P_1^1 - Peg^1P_1^1$. If, now, we cut out two pieces of wood, one having its side shaped like the curve PeP_1^1 and pivoted at A, while the other is shaped like PfP_1^1 and pivoted at B; then from what has been said, the former may be used to drive the latter, and the motion will be the same as that produced by the rolling of the two pitch circles together, hence these shapes will be the proper ones for the *profiles* of gear teeth.

The curves Pe, Pf, P^1g and P^1h, which are produced by the rolling of one circle inside or outside of another are called *cycloidal* curves, the two Pe and P^1h being known as hypocycloids, since they are formed by the describing circle rolling inside the pitch circle, while the two curves Pf and P^1g are known as epicycloidal curves, in this case lying outside the pitch circles. Gears having these curves as the profiles of the teeth are said to have cycloidal teeth (sometimes erroneously called epicycloidal teeth), a form which is in very common use. So far we have only drawn one side of the tooth, but it will be evident that the other side is simply obtained by making a tracing of the curve PeP^1_1 on a piece of tracing cloth, with centre A also marked; then by turning the tracing over and bringing the point A to the original centre A, the other side of the tooth on the wheel ecg may be pricked through with a needle. The same method is employed for the teeth on wheel fch.

Nothing has so far been said of the sizes of the describing circles, and, indeed, it is evident that any size of describing circle, so long as it is somewhat smaller than the pitch circle, may be