Let x = one of the segments determined on the base by the perpendicular; then

$$x = b + \sqrt{\frac{4\,h^2\,d^2}{b^2 - d^2} + d^2} \quad \text{N. S.}$$

$$\text{make } b^2 - d^2 = y^2 \qquad \text{(by formula 5),}$$

$$\text{then } x = b + \sqrt{\frac{4\,h^2\,d^2}{y^2} + d^2} = b + \sqrt{\frac{4\,h \times h}{y} \times \frac{d \times d}{y} + d^2}$$

$$\text{make } \frac{4\,h \times h}{y} = t\text{; and } \frac{d \times d}{y} = s \text{ (by formula 2),}$$

$$\text{then } x = b + \sqrt{t\,s + d^2}$$

$$= b + \sqrt{r^2 + d^2} \qquad \text{(See Problem VII.)}$$

$$= b + v \quad \text{(by formula 4).} \qquad \text{G. S.}$$

N.B.—In this case, because the Graphical Solution requires many transformations, the numerical solution is much preferable.

PROBLEM XXXII.

Having given the three sides a, b, c, of a triangle, to find the radius r of the inscribed circle.

$$r = \frac{a\sqrt{b^2 - \dfrac{(a^2 + b^2 - c^2)^2}{4\,a^2}}}{a + b + c} \quad \text{N. S.}$$

PROBLEM XXXIII.

To determine a Right-Angled Triangle, having given the hypotenuse a, and the radius r of the inscribed circle.

Let b and c be the two other sides.

$$b = r + \frac{a}{2} + \tfrac{1}{2}\sqrt{a^2 - 4\,a\,r - 4\,r^2} \text{ N. S.}$$

$$c = r + \frac{a}{2} - \tfrac{1}{2}\sqrt{a^2 - 4\,a\,r - 4\,r^2} \text{ N. S.}$$

PROBLEM XXXIV.

Having given the two equal sides a of an Isosceles triangle, and the base b, to find the radius r of the inscribed circle.

$$r = \frac{\dfrac{b}{2}\sqrt{a^2 - \dfrac{b^2}{4}}}{a \times \tfrac{1}{2}\,b} \qquad \text{N. S.}$$