STRATFORD.—Thunder with rain 20th. Lightning and thunder with rain 12th, 24th, 25th, Frost 1st, 2nd, 3rd, 7th, 9th. Wind storms 9th, 25th. Rain 12th, 15th, 16th, 17th, 20th, 24th, 25th, 26th, 31st. Excess of temperature over average of May thirteen years + 1°17.

HAMILTON.—Lightning 12th. Lightning and thunder with rain 20th, 31st. Frost 9th. Rain 12th, 16th, 20th, 31st.

SIMCOE.—Rain 12th, 16th, 17th, 18th, 20th, 25th, 31st. No observations here on 1st and 2nd owing to removal of instruments.

WINDSOR.—Lightning 12th, 28th. Hail 17th. Lightning with thunder 24th. Lightning and thunder with rain 29th. Meteor in E. towards H. and one in N. towards H., 10th. Meteor through *Draco* and *Polaris* towards H. at N.W. on 12th. Solar and Lunar halo 22nd. Solar halo 23rd and 24th. Frost 1st, 3rd, 7th, 19th. Wind storms 2nd, 25th. Rain 4th, 15th, 17th, 19th, 20th, 29th.

V. Mathematical Department.

To the Editor of the Journal of Education.

Toronto, 22nd June, 1874.

SIR,—I herewith send you solutions of the questions in Algebra and Natural Philosophy, which were proposed to candidates for First Class Certificates at the recent examination of the students of the Normal School.

I may state that Mr. Birchard, to whom a First Class Certificate of the highest grade was awarded, solved correctly all the questions in Natural Philosophy; and, instead of writing out solutions of my own, I send you those which he gave. I asked him to re-write his paper, making no material change, but only extending abbreviations, and (as I wished to dispense with diagrams) making such statements, not necessary where the diagram was exhibited, as might render his work intelligible in the absence of the diagram. The paper will be felt, I think, to be a creditable production.

The solutions of the questions in Algebra have also been prepared by Mr. Birchard, though these solutions do not exhibit his work done in the hall.

As I understand that the solutions of the first-class questions in Algebra and Natural Philosophy, which appear from time to time in the *Journal*, are extensively studied, I have, in order to prevent misapprehension, added a note on the first question in the Natural Philosophy paper of July, 1873.

I am, Sir,
Your obedient servant,
GEORGE PAXTON YOUNG.

NORMAL SCHOOL EXAMINATION, JUNE, 1874.

Solution of First Class Questions in Algebra and Natural Philosophy.

ALGEBRA.

1. (*a*) Substitute vy for x; then

$$v^2y^2+6vy^2+27=0\text{ , or , }y^2=-\frac{27}{v^2+6v}$$

$$y^2-2vy^2-16=0,$$

$$\text{or, }y^2=\frac{16}{1-2v}.$$

$$\frac{-27}{v^2+6v}=\frac{16}{1-2v}.\text{ Therefore, \&c.}$$

(*b*) $\sqrt{3x^2+x-1}=9x^2+3x-5$; re-arrange this, then $3(3x^2+x-1)-\sqrt{3x^2+x-1}=2$; this gives $\sqrt{3x^2+x-1}=-\frac{2}{3}$ or 1. Therefore, &c.

(*c*) $x^4+2x^3+2x^2+2x+1=0.$

$$x^2+2x+2+\frac{2}{x}+\frac{1}{x^2}=0.$$

$$\left(x^2+2+\frac{1}{x^2}\right)+2\left(x+\frac{1}{x}\right)=0.$$

$$\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}+2\right)=0.$$

$$x+\frac{1}{x}=0\text{ and }x+\frac{1}{x}+2=0.$$

$$x=\pm\sqrt{-1}\text{ or }x=-1,$$

hence the values of x are $\text{I}\sqrt{-1}$, -1 and -1.

It will be observed that two of the roots are equal.

[Instead of using the general method for solving a recurring equation, it would have been simpler, in this particular case, to have proceeded as follows:

$$x^4+2x^3+2x^2+2x+1=(x^4+2x^3+x^2)+(x^2+2x+1)$$
$$=(x^2+2x+1)(x^2+1).$$

Therefore the roots of the given equation are the roots of the equations,

$$x^2+2x+1=0$$
$$\text{and, }x^2+1=0.$$

G. P. Y.]

2. Let $x=$ number of minute spaces passed over by the hour hand after 7 o'clock. Then $12x=$ spaces passed over by minute hand; and since the minute hand is only two minutes behind the hour hand, we have $x+35-2=12x$, or, $x=3$; and hence the time is 7h. 36m.

Now, in order to give 11 spaces, the minute hand must pass over 12 spaces; and to find the true time it takes to do this state thus, $719 : 12 :: 720 : 12\frac{12}{719}$ ∴ the true time is $36+12\frac{12}{719}=48\frac{12}{719}$ minutes after 7 o'clock.

3. Let x and y be the extremes, then $\frac{2xy}{x+y}$ is the mean.

$$\therefore\ x2+\left(\frac{2xy}{x+y}\right)+2y^2=7 \qquad (1)$$

$$x\times\frac{2xy}{x+y}\times y=\frac{x+y}{2} \qquad (2)$$

Simplifying (2), we get $2xy=x+y$ (3)

Thus equation (1) becomes $x^2+1+y^2=7$ (4)

Squaring (3) and substracting (4), we get $4x^2y^2-2xy=6$, or $xy=\frac{3}{2}$ or -1; it is now easy to get the values of x and y which give the following series: $\frac{1}{2}(3+\sqrt{3})$, $1\frac{1}{2}(3-\sqrt{3}$, or $-1+\sqrt{2}$, 1, $-1-\sqrt{2}$.

4. $\left.\begin{array}{l}(m+1)^2+a(m+1)+b=0\\(m-1)^2+a(m-1)+b=0\end{array}\right\}$

Hence $m+1$ and $m-1$ must be the roots of the equation, $y^2+ay+b=0$; but the roots of this equation are $\frac{1}{2}(-a+\sqrt{a^2-4b})$ and $\frac{1}{2}(-a-\sqrt{a^2-4b})$; and the roots of the given equation, $x^2+2ax+4b=0$, are $-a+\sqrt{a^2-4b}$ and $-a-\sqrt{a^2-4b}$ which are just twice the former roots; but the roots of the former equation were $m+1$ and $m-1$, therefore the roots of the latter are $2(m+1)$ and $2(m-1)$.

[Would it not have been more direct to have reasoned as follows:

Put the equation, $x^2+2ax+4b=0$, in the form $\left(\frac{x}{2}\right)^2+a\left(\frac{x}{2}\right)+b=0$. Then, by the given conditions, $m+1$ and $m-1$ satisfy this equation; that is, the two values of $\frac{x}{2}$ are $m+1$ and $m-1$; therefore the two values of x are $2(m+1)\times 2(m-1)^2$.

G. P. Y.]

5. Let the roots of the equation $x^2+px+q=0$ be a and b; then $a+b=-p$, and $ab=q$.

Also, from the question, $a^3+b^3=4(a+b)^3$;

or, $(a+b)(a^2-ab+b^2)=4(a+b)^3$

Substituting the values of $a+b$ and ab; we get $-p(p^2-3q)=-4p^3$; transposing and reducing, we get $3p(p^2+q)=0$; hence either p or $p^2+q=0$.

[Otherwise thus:

$$4(a+b)^3=a^3+b^3=(a+b)^3-3ab(a+b).$$
$$\therefore -4p^3=-p^3+3pq$$
$$\therefore 3p(p^2+q)=0.\text{ Therefore, \&c.}$$

G. P. Y.]

6. Let a and b be the roots of the equation, $x^2+mx+n=0$; and A and B the roots of $x^2+Mx+N=0$. Then $(a-b)=\sqrt{(m^2-4n)}=d$; and $A-B=\sqrt{(M^2-4N)}=D$.

Then $\frac{d^2}{D^2}=\frac{m^2-4n}{M^2-4N}=\frac{n}{N}$ by question.

Now, since the numerator and denominator $\frac{m^2-4n}{M^2-4N}$ are of the same form [not very well expressed, G. P. Y.] and $\frac{4n}{4N}=\frac{d^2}{D^2}$ therefore also $\frac{m^2}{M^2}=\frac{d^2}{D^2}$ hence $\frac{m^2}{M^2}=\frac{n}{N}$.

7. Let $x=$ speed of the train from A in miles per hour; and $y=$ speed of the train from B in miles per hour.

$$\frac{12}{x}-\frac{10\frac{1}{4}}{60}=\text{time before second train starts,}$$