TRIGONOMETRY. (1892)

SENIOR LEAVING

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1\. *(a)* Birchard's Trigonometry, "The Trigonometrical Angle," articles 12, 18, 20.

(b) "Measurement of Angles," articles 25, 26, and 36, 37, 38.

(c) Let a be the length of arc which subtends angle at the centre of 2.5 radians. Whole angle at centre is 2π radians

$$\therefore \frac{a}{c} = \frac{2.5}{2.\pi},$$

where c = circumference of circle

$$\therefore c = 2.\pi r$$

$$= 8\pi$$

$$\frac{a}{8\pi} = \frac{2.5}{2.\pi}$$

$$\frac{a}{4} = 2.5$$

$\therefore a = 10$ ft. *Ans.*

Let b be the length of arc which subtends an angle at the centre of 70°.

$$\therefore \frac{b}{8\pi} = \frac{70}{360}$$

$\therefore b = 4\frac{8}{9}$ ft. *Ans.*

2\. *(a)* Birchard's Trigonometry, chap. 3, art. 43.

Prove, $\sec A + \sin^2 A + \cot^2 A = \mathrm{cosec}^2 A + \cos^2 A (\sec^2 A - 1)$

$\sec A + \sin^2 A + \cot^2 A - \mathrm{cosec}^2 A - \cos^2 A (\sec^2 A - 1) =$

$$\sec A + \sin^2 A + \frac{\cos^2 A}{\sin^2 A} - \frac{1}{\sin^2 A} - \frac{\cos^2 A}{\cos^2 A} + \cos^2 A =$$

$$\sec A + \sin^2 A + \cos^2 A + \frac{\cos^2 A - 1}{\sin^2 A} - \sec A =$$

$$1 - \frac{\sin^2 A}{\sin^2 A} =$$

$$1 - 1 = 0$$

$\therefore \sec A + \sin^2 A + \cot^2 A - \mathrm{cosec}^2 A - \cos^2 A(\sec^2 A - 1) = 0$

$\therefore \sec A + \sin^2 A + \cot^2 A = \mathrm{cosec}^2 A + \cos^2 A(\sec^2 A - 1)$

(b) Take your two axes at right angles to one another.

On X-axis mark off OL containing 13 units; on Y-axis mark off OM containing 5 units. With O as centre and OL as distance describe a circle. Through M draw NP parallel to SOR, meeting circumference at N and P, join OP, ON, draw PR and NS perpendicular to SR.

$\angle ROP$ and $\angle RON$ are angles having sin $\frac{5}{13}$

2\. *(b)* Because $OM = RP = NS = 5$ units, and $OL = OP = ON = 13$ units

$$\sin ROP = \frac{RP}{OP} = \frac{5}{13}$$

$$\sin RON = \frac{NS}{ON} = \frac{5}{13}$$

$\therefore$ ROP, RON are the angles of which the sine is $\frac{5}{13}$

First take $\angle$ to be $\angle ROP$.

$$OR^2 = OP^2 - RP^2$$

$$= 13^2 - 5^2$$

$$= 144$$

$$\therefore OR = 12$$

$$\sin(90^\circ + d) = \cos d = \frac{12}{13}$$

$$\cos(90^\circ + d) = -\sin d = -\frac{5}{13} \text{ etc.}$$

$$\sin(180^\circ - d) = \sin d = \frac{5}{13}$$

$$\cos(180^\circ - d) = -\cos d = -\frac{12}{13} \text{ etc.}$$

Next take d to be $\angle RON$.

The $\triangle OSN$ is geometrically equal in all respects to $\triangle OLP$; but trigonometrically $OR = -OS$.

But $OR = -12$.

$\therefore OS = -12$