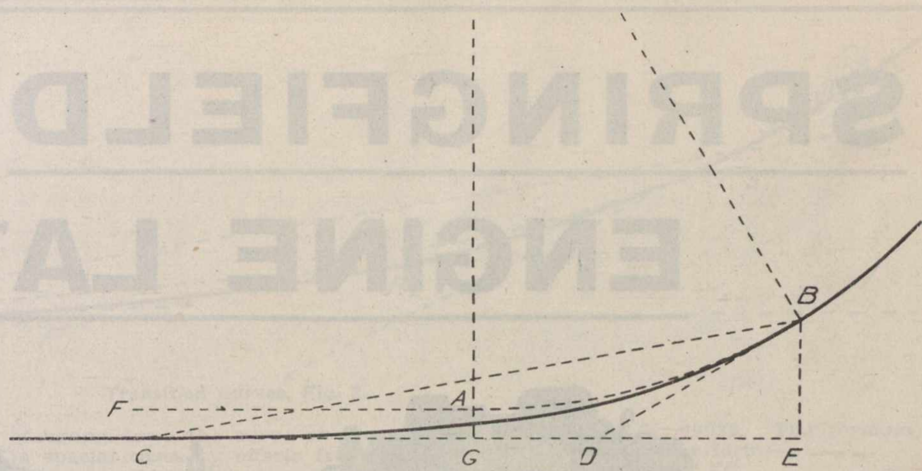
this point on, while in the other we shall continue it to P_{10}.

Now, as to the proper chord length. It has been found by experiment that in order to avoid any ill effects from an abrupt change in the position of the car body, from upright to inclined, the elevation of the outside rail should not gain on that of the inside more than about ¾ of an inch per second. At 60 miles an hour a train will travel 90 ft. (88 ft. exactly) in this time, and with our ordinary rule for elevating at ½ inch per degree of curvature, ¾ inch would correspond to a 1° 30′ curve. Proportionately, the distance for a 1° curve would be 60 ft., and this would be our chord length for a 60 mile speed. But this speed is not a safe one on curves of more than 3° or thereabouts. And as the longer chord length and resulting transition throws the curve inside its tangents much more than the short, consequently lessening its efficiency in avoiding an obstacle, it is desirable and quite permissible to use a shorter chord length in connection with the sharper curves and a corresponding less elevation of outer rail.

Following out this reasoning the writer suggested some years ago that inasmuch as the centrifugal force varied as to the square of the speed, and the permissible speed varied about in the inverse ratio of the degree of curvature, the proper formula for elevation should be one involving the square root of the degree of curvature, and that if E. represented the elevation in inches and D. the degrees of curvature, then the simple formula $E. = \sqrt{D}$ would give results not far from the best practice on curves between 1° and 10°. Engineering News commented very favorably on the suggestion, and it is believed to be in common use.

In view of these considerations, but in order to avoid unnecessary complication,



Transition Curves, Fig. 1.

three standard chord lengths, 60′, 45′ and 30′ will be used, the first for curves up to 3° or 4°, the second to 7°, and the third from 7° to 10°. Main line curves sharper than 10° are seldom used, and when they are resorted to are necessarily run over at very low speeds, and considerations other than centrifugal force, properly so called, make it desirable that the super-elevation should be still further reduced, hence, while it is still desirable to insert a transition of some kind, it is permissible to make it very short indeed. Our rule would give for a 16° curve, for instance, 4 ins. of elevation, while the older rule of ½ in. per degree would give 8 inches. This last is quite inadmissible, and the first more than will be found in good practice, and for the following reasons:—

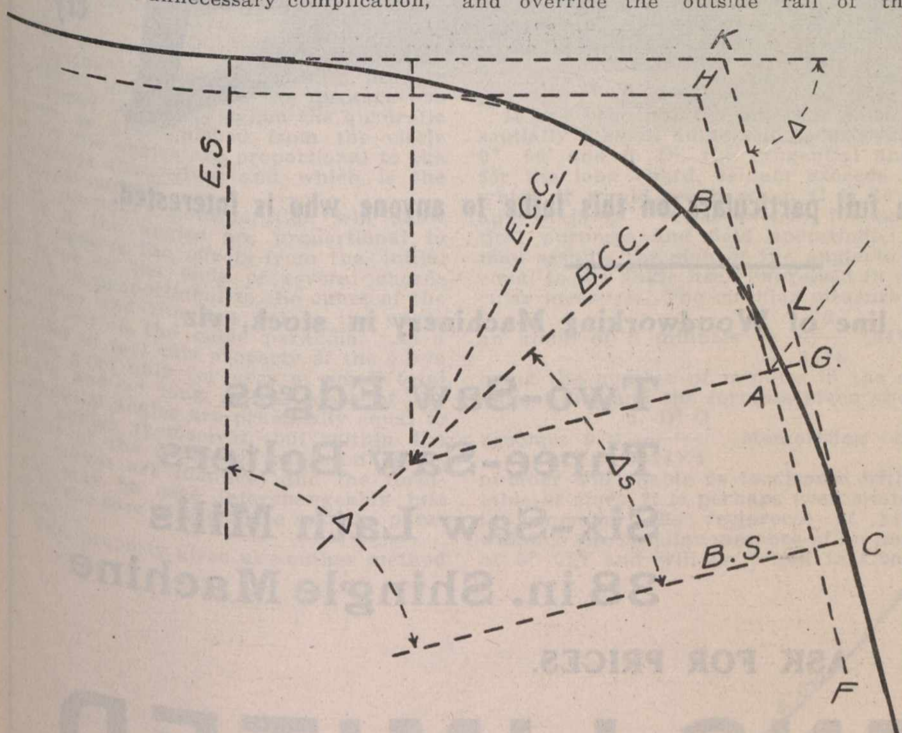
Super-elevation counteracts the dynamical centrifugal force due to velocity, but there is a tendency to thrust against and override the outside rail of the curve which, while it acts in the same direction, is quite independent of the centrifugal force, and is due to the obliquity of the car axles to the radius of the curve. At low speeds and on very sharp curves it is very much the most important component of the two, and no moderate amount of super-elevation will counteract it to any considerable extent, but rather the reverse, for the wheel is held on the track by contact of the outside flange against the head of the rail, and is constantly trying to climb up over it, and only prevented from doing so by the weight of the car upon its journal. Decrease this weight by tilting the car inwards until we throw all the weight on the inside rail, and the outside wheel will inevitably climb over and the car be derailed. It follows that on a sharp curve with excessive elevation it is safer to move at a speed sufficient to generate sufficient centrifugal force to equalize the weight on the two wheels, than it is to crawl around it, and actual experience proves the truth of this apparent paradox.

To return to our transition curve. We have established a 60 ft. chord for curves up to 3°, and for this curvature we shall have three chords terminating at points P_1, P_2, P_3.

Refer to Figure 2.—The mean curvature is 1° 30′, and the total length 180 ft., giving a total deflection 2° 42′ or 162′ and a tangential angle for P_3 of $\frac{162}{3}$ or 54′. As it has been shown that the tangential angle to any point is proportional to the square of the distance of that point from the beginning of the transition curve, then the corresponding angles for the intermediate points P_1 and P_2 will be $(⅓)^2$ or 1-9 and $(⅔)^2$ or 4-9 respectively of the tangential angle at P_3, thus giving an angle of 6′ at P_1 and 24′ at P_2. We have then for the 60 ft. chord transition the series of tangential angles:—

$$6 \times 1^2 = 6'; \quad 6' \times 2^2 = 24'; \quad 6' \times 3^2 = 54'.$$

To run the curve by the transit we set up on the initial point P. and turn off these successive angles from the tangent produced. Arrived at P_3, the end of the transition for a 3° curve, we put in a hub. Setting up over it we sight back at P. and turn off for the tangent at P_3 not 0° 54′, but double this or 1°, 48′ from which tangent we run in the 3° circular curve in the ordinary way. Arrived at the other end we simply reverse the process, or we may proceed as follows, still referring to fig. 2 At P_3 the 3° curve produced would swing inside the transition and leave it at exactly the same rate that the transition left the tangent at P. Our angles from the tan-



Transition Curves, Notation Diagram.

RECAPITULATION OF SYMBOLS AND FORMULAS.

Offset of Main Curve = O = A.G.
Length of Spiral = S = B.C.
Total Deflexion Angle of Spiral = Δs.
Total Deflection Ang e = Δ
Chord Length = c.
Deflexion Angle to 1st Chord Point = d. = $\frac{c}{10}$

Deflexion Angle to any Chord Point = dn = $\frac{c}{10} \times n^2$
Degree of Constant Curve = D.
Sub Tangent of Constant Curve = T. = A.H.
Sub Tangent of Spiralled Curve = T_s = K.C.
Correction for Sub Tangent = O tan. ½ Δ = G.L.