## Algebra.

## EXERCISE I.

Factoring is the most important section of the algebra for a P.S.L. pupil to understand. We shall in this column endeavor to give some help to our readers in this part of the work. (Answers in The Canadian Téacher.)

## FACTORING BY PARTS.

Suppose we have $a^{2}+a b+a c+b c$ to factor. We at once see that a will come out of 1 and 3 , leaving $a(a+c)$ and $b$ will come out of 2 and 4 , leaving $b(a+c)$. So that $a^{2}+a b+a c+b c=a(a+c)+b$ $(a+c)$. Now, we have reduced the expression to one of two terms, each of which is divisible by $a+c$. Therefore $a(a+c)+b(a+c)=(a+c)(a+b)$, or the factors of $a^{2}+a b+a c+b c$ are $(a+c)(a+b)$.

The following problems may be factored by this method:

## EXERCISES

1. $a^{2} c^{2}+a c d+a b c+$ bd.
2. $2 x+c x+2 c+c^{2}$.
3. $5 a+a b+5 b+b^{2}$.
4. $a^{2}-a c+a b-b c$.
5. $a^{2}+3 a+a c+3 c$.
6. $x^{2}-a x+5 x-5 a$.
7. $a b-b y-a y+y^{2}$.
8. $a x-b x-a z+b z$.
9. $\mathrm{pr}+\mathrm{qr}-\mathrm{ps}-\mathrm{qs}$.
10. $\mathrm{mx}-\mathrm{my}-\mathrm{nx}+\mathrm{ny}$.
II. $m x-m a+n x-n a$.
11. $2 a x+a y+2 b x+b y$.
12. 3 ax $-b x-3 a y+b y$.
13. $6 x^{2}+3 x y-2 a x-a y$.
14. $m x-2 m y-n x+2 n y$.
15. $a x^{2}-3 b x y-a x y+3$ by ${ }^{2}$.
16. $x^{2}+m x y-4 x y-4$ my ${ }^{2}$.
17. $\mathrm{ax}^{2}+\mathrm{b} \mathrm{x}^{2}+2 \mathrm{a}+2 \mathrm{~b}$.
18. $x^{2}-3 x-x y+3 y$.
19. $2 x^{4}-x^{3}+4 x-2$.
20. $3 x^{3}+5 x^{2}+3 x+5$.
21. $x^{4}+x^{3}+2 x+2$.
22. $y^{3}-y^{2}+y-1$.
23. $a x y+b c x y-a z-b c z$.
24. $\mathrm{f}^{2} \mathrm{x}^{2}+\mathrm{g}^{2} \mathrm{x}^{2}-\mathrm{ag}^{2}-$
$\mathrm{at}^{2}$.
25. $2 a x^{2}+3 a x y-2 b x y-$ $3{ }^{2} y^{2}$.
26. $a m x^{2}+b m x y-a n x y$ $-b n y{ }^{2}$.
27. $a x-b x+b y+c y-c x$ -ay.
28. $a^{2} x+a b x+a c+a b y+$ $b^{2} y+b c$.

## EXERCISE II.

Below we give two special methods in factoring and fourteen exercises on them.
I.

By a slight modification some expressions admit of being written in the form of the difference of two squares, and may then be resolved into factors :

1. Resolve into factors $x^{4}+x^{2} y^{2}+y^{4}$.

$$
\begin{aligned}
x^{4}+x^{2} y^{2}+y^{4} & =\left(x^{4}+2 x^{2} y^{2}+y^{4}\right)-x^{2} y^{2} \\
& =\left(x^{2}+y^{2}\right)^{2}-(x y)^{2} . \\
& =\left(x^{2}+x y+y^{2}\right)\left(x^{2}-x y+y^{2}\right) .
\end{aligned}
$$

2. Reso've into factors $x^{4}-15 x^{2} y^{2}+9 y^{4}$.

$$
\begin{aligned}
x^{4}-15 x^{2} y^{2}+9 y^{4} & =\left(x^{4}-6 x^{2} y^{2}+9 y^{4}\right)-9 x^{2} y^{2} . \\
= & \left(x^{2}-3 y^{2}\right)^{2}-(3 x y)^{2} . \\
= & \left(x^{2}-3 y^{2}+3 x y\right)\left(x^{2}-3 y^{2}-3 x y\right) \\
& \quad 11 .
\end{aligned}
$$

The quotient of $a^{3}+b^{3}+c^{3}-3 a b c$ by $a+b+c$ is $a^{2}+b^{2}+c^{2}-b c-c a-a b$.

This result is important, and should be carefully remembered. We may note that the expression on the left consists of the sum of the cubes of three quantities, a, b, c, diminished by three times the product abc. Whenever an expression admits of a similar arrangement, the sbove formula will enable us to resolve it into factors.
3. Resolve into factors $a^{3}-b^{3}+c^{3}+3 a b c$.
$a^{3}-b^{3}+c^{3}-3 a b c=a^{3}+(-b)^{3}+c^{3}-3 a(-b ; c$.

$$
\begin{aligned}
= & (a-b+c)\left(a^{2}+b^{2}+c^{2}+b c\right. \\
& -c a+a b
\end{aligned}
$$

An additional method of factoring these will be found in H.S.A., page 62, section 105.

## EXERCISES.

| 1. $x^{4}+16 x^{2}+256$. | 9. $a^{3}+(a+b) a x+b x^{2}$. |
| :--- | :--- |
| 2. $81 a^{4}+9 a^{2} b^{2}+b^{4}$. | 10. $\left(2 x^{2}-3 a^{2}\right) y+\left(2 a^{2}\right.$ |
| 3. $x^{4}+y^{4}-7 x^{2} y^{2}$. | $\left.-3 y^{2}\right) x$. |
| 4. $9 x^{4}+4 y^{4}+11 x^{2} y^{2}$. | 11. $b^{3}+c^{3}-1+3 b c$. |
| 5. $x^{4}-19 x^{2} y^{2}+25 y^{4}$ | 12. $8 a^{3}+27 b^{3}+c^{3}-$ |
| 6. $16 a^{4}+b^{4}-28 a^{2} b^{2}$. | $18 a b c$. |
| 7. $x^{2} y+3 x^{2}-3 x^{3}-y^{3}$. | 13. $x^{7}+x^{4}-16 x^{3}-16$. |
| 8. $4 \mathrm{mn}^{2}-20 n^{3}+45 \mathrm{~mm}^{2}$ | 14. $16 x^{7}-81 x^{3}-16 x^{4}$ |
| $\quad-9 m^{3}$. | +8 r. |

## Bookkeeping.

Below we give the Public School Leaving bookkeeping paper for 1895. In our next issue it will be fully solved.

Work out the following set in single entry using Daybook, Cashbook and Ledger :

Toronto, May 1, 1895, rented a store from J. Shield at \$20 per month, and began business. At time of commencement I owed Turcotte \& Co. $\$ 21$ and had $\$ 250$ in cash, merchandise worth $\$ 345$, a note at 60 days for $\$ 73$, dated Jan. 15 th 1895, against J. Bell, and Tarbutt \& Son owe me $\$ 1625$ on contra account.

May 2. Sold J. Larneek, on account, 37 yards velveteen at $\$ 2.25$ per yard.

May 3. Paid Turcotte \& Co., on account, \$12.75.

May 6. Tarbutt \& Son gave me an order on W. Williams to pay their account in full.

May 7. Gave away $\$ 5$, lost $\$ 5$.
May 8. Bought merchandise (invoice $\$ 237.84$ ) from J. Macdonald \& Co., paid cash \$100, and gave my note for balance. Spent $\$ 1.35$ on repairs.

May 10. Sold Turcotte \& Co. 100 barrels apples at $\$ 4$ and groceries amounting to $\$ 21.40$.

May. 11. Paid for putting up shelves in store $\$ 12$ 40, postage, etc., $\$ 3$.

May 13. W. Williams paid by cheque on bank the order of May 6th.

Note.-No inventory being given we may take for granted that all the merchandise has been sold

