7. Solve the equations

$yz + zx + xy = 3.$

$yz(y+z) + zx(z+x) + xy(x+y) = 3.$

$yz(y^2+z^2) + zx(z^2+x^2) + xy(x^2+y^2) = 3.$

7. The given equations may be written in the following form:

$$yz + zx + xy = 3. \qquad (1)$$

$$(x+y+z)(yz+zx+xy) = 3. \qquad (2)$$

$$(zx+xy+yz)(x^2+y^2+z^2) - xyz\,(x+y+z) = 3. \qquad (3)$$

$\therefore$ from (1), (2) becomes $(x+y+z) - xyz = 1.$

$\therefore\ x+y+z-1 = xyz \quad (4)$

$\therefore$ substituting from (1) and (4) in (3),

$3(x^2+y^2+z^2) - (x+y+z)^2 + (x+y+z) = 3.$

$2x^2+2y^2+2z^2-2xy-2xz-2yz + (x+y+z) = 3.$

$2(x+y+z)^2 - 4(yx+yz+zx) + (x+y+z) = 3.$

from (1) $2(x+y+z)^2 + (x+y+z) - 15 = 0.$

$$x+y+z = -\frac{1\pm\sqrt{121}}{4} = -\frac{1\pm 11}{4} = -3 \text{ or } \frac{5}{2}$$

Taking $(x+y+z) = \frac{5}{2}$, $\therefore$ from (4) $xyz = \frac{3}{2}$

$$\therefore\ -yz = \frac{3}{2x}.$$

(1) $yz + x(y+z) = 3$, $\therefore\ \frac{3}{2x} + x\left(\frac{5}{2} - x\right) = 3.$

$$2x^3 - 5x^2 + 6x - 3 = 0.$$

$$(x-1)(2x^2 - 3x + 3) = 0.$$

$$\therefore\ x = 1, \text{ or } x = \frac{3\pm\sqrt{-15}}{4}.$$

and values may be obtained from y and z.

8. If $\sqrt{x+a+b} + \sqrt{x+c+d} = \sqrt{x+a-c} + \sqrt{x-b+d}$, then $b+c=0$.

8. $\sqrt{x+a+b} - \sqrt{x+a-c}$

$$= \sqrt{x-b+d} - \sqrt{x+c+d}$$

$$\frac{(x+a+b)-(x+a-c)}{\sqrt{x+a+b}+\sqrt{x+a-c}} = \frac{(x-b+d)-(x+c+d)}{\sqrt{x-b+d}+\sqrt{x+c+d}}$$

$$\therefore\ \frac{b+c}{\sqrt{x+a+b}+\sqrt{x+a-c}} = \frac{-(b+c)}{\sqrt{x-b+d}+\sqrt{x+c+d}}$$

$\because\ b+c=0.$

SOLUTIONS TO PROBLEMS.

BY WILBUR GRANT, C. I., TORONTO.

1. A hollow cylinder closed at both ends is filled with water and held with its axis horizontal; if the whole pressure on its surface, including the plane ends, be three times the weight of the water, compare the height and diameter of the cylinder.

1. Let r = radius of cylinder,

" h = height "

then area of each end of cylinder $= \pi r^2$,

" surface " $= 2\pi r \cdot h$.

Depth of *C. G.* of cylinder below highest point $= r$, fluid pressure on surface of cylinder

$= \left\{2\pi r \cdot h + 2\pi r^2\right\} r \cdot \frac{1000}{1728}$ oz.

weight of water in cylinder $= \pi r^2 h \cdot \frac{1000}{1728}$ oz.

$\left\{2\pi r \cdot h + 2\pi r^2\right\} r \cdot \frac{1000}{1728} = \pi r^2 h \frac{1000}{1728}$.

$h = 2r$ = diameter.

2. Find whole pressure on an equilateral triangle immersed in water whose side is 8 feet and vertex 10 inches below the surface, the base being horizontal.

2. Area of triangle $= 16\sqrt{3}$ sq. ft.

$= 16 \times 1{\cdot}7320$ sq. ft. nearly,

$= 27{\cdot}712$ sq. feet.

Depth of *C. G.* of triangle below surface of fluid $= \left(\frac{2}{3} \text{ of } 4\sqrt{3} + \frac{5}{6}\right)$ ft.

fluid pressure on triangle $= 16\sqrt{3}$

$$\left(\frac{16\sqrt{3}+5}{6}\right) \times 1000 \text{ oz.}$$

$= 151093\frac{1}{2}$ oz. nearly.

3. A pipe 15 feet long closed at the upper extremity is placed vertically in a tank of the same height; the tank is then filled with water; if the height of the water-barometer be 33 feet 9 inches, determine how high the water will rise in the pipe.

3. Let x = height of water in tube,

" a = area of cross section of tube,

$$\frac{15}{15-x} = \frac{a \cdot \overline{15-x} \cdot 1000 + 33\tfrac{3}{4} \times a \times 1000}{33\tfrac{3}{4} \times a \times 1000}$$

$$= \frac{\overline{15-x} + 33\tfrac{3}{4}}{33\tfrac{3}{4}}$$

$$\frac{x}{15-x} = \frac{15-x}{33\tfrac{3}{4}}$$

$$x = \frac{255}{8} \pm \frac{255}{8} = 60 \text{ or } 3\tfrac{3}{4} \text{ ft.}$$

$\therefore\ 3\frac{3}{4}$ ft. height in tube.