THE MOMENT DIAGRAM AND ITS RELATION TO THE REINFORCEMENT IN A CONCRETE BEAM.*

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In the past the design of reinforced concrete beams has involved some intricate problems relative to the proper placing of the steel reinforcement in the body of the beam. Extensive analytical methods have been resorted to, or a series of graphic constructions have been necessary, to determine the relative position of the component parts of the reinforcing material.

In the present paper it is proposed to set forth a method of placing the reinforcement for both bending and shearing resistance entirely from the moment diagram. To explain the method it is deemed necessary to review the principles upon which later operations are based, after which its application will be made to a specific design.

Consider a simple beam, uniformly loaded, as in Fig. 1. Let two transverse vertical sections, (1) and (2), be passed through the beam, at distances x_1 and x_2 from the left support. Then from mechanics it may be shown that

$$M_2 - M_1 = \frac{V_1 + V_2}{2}(x_2 - x_1)$$

Whence

$$V = \frac{M_2 - M_1}{s} \qquad (1)$$

in which V equals the average vertical shear on the portion of the beam $(x_2 - x_1) = s$.

Or,—The difference in moment between any two points along a beam is equal to the product of the average shear over the distance between the points, and that distance.

For loads concentrated at points along a beam this law is not strictly true, unless in each case the concentration occurs at a point midway between the transverse sections chosen; but in the case of "concentrated" loadings by beams cast against the girders in concrete construction, and even by loadings on slabs transmitted finally to the girder, the concentration may not be sharply defined, and there is no determinate law of shear variation over such a region. Moreover, as this discussion will show later, the distance s is relatively small where shear is large. Within the limits of actual conditions in reinforced concrete construction, therefore, the above statement may be considered very approximate for the beam loaded with concentrated loads.

From the discussions given in Turneaure and Maurer, "Principles of Reinforced Concrete Construction," pp. 109 and 223, the amount of tensile stress required of a vertical stirrup to resist the shear stress is

$$\frac{Vs}{jd} \qquad (2)$$

in which V is the average vertical shear over the portion s of the beam. The idea of maximum shear intensity on a vertical section of the beam is retained in the above expression.

Since the tensile strength of the stirrup is relied upon to carry the above force, when s is the space between adjacent stirrups, the total strength of the stirrup, $a_s f_s$, would be equal to eq. (2), or

$$a_s f_s = \frac{Vs}{jd}$$

from which

$$V = \frac{a_s f_s jd}{s} \qquad (3)$$

which is the value of the average vertical shear over the portion s of the beam, to be resisted by the stirrup, in terms of the strength of the stirrup and certain dimensions of the beam.

In eq. (1) we have an expression of this average shear in terms of the change in moment along the portion s; so that by substituting in the above, we have at once,

$$M_2 - M_1 = a_s f_s jd \qquad (4)$$

Let it be considered that the portion s be so chosen that section (1), Fig. (1), lies over the left abutment, at which point the moment M_1 is zero. (In continuous girder design section (1) may be considered as lying at a point of inflection, since the moment at that place is

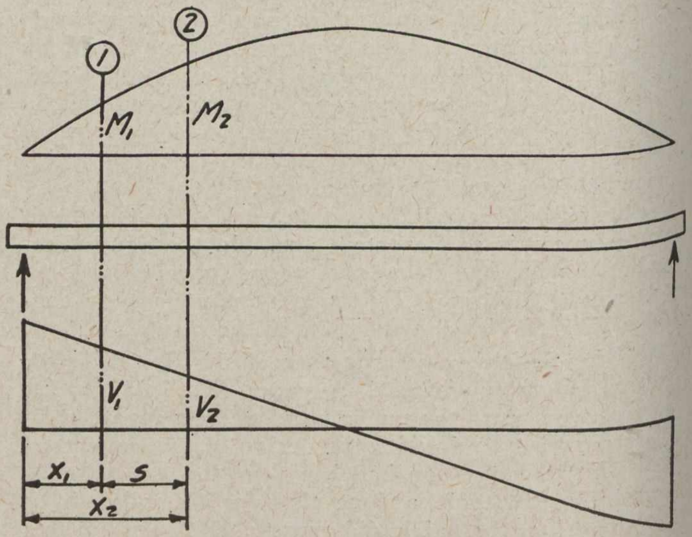

Fig. 1.

zero.) Let the moment increment from zero to M_2 be called M_I. Then from eq. (4),

$$M_I = a_s f_s jd \qquad (5)$$

It is to be noted that this moment increment represents the value of the adopted stirrup to resist the vertical component of diagonal tension over the portion s of the beam. Again, it should be clear that for any given stirrup of area a_s and of fiber stress f_s, the moment increment M_I varies directly with jd, a value dependent upon the characteristics of the beam; and that for a given beam and the given stirrup, the moment increment is a constant, irrespective of where the region s is chosen along the beam.

From values given in tests published in "Principles of Reinforced Concrete Construction," it may be noted that the safe working shear stresses are about three times as great in a reinforced concrete beam as when the beam is not reinforced. We may say, therefore, that the concrete will be permitted to carry one-third of the shear, and the remainder will be cared for by the reinforcement. Eq. (5) then becomes (6)

$$M_I = 1.5\, a_s f_s jd \quad \text{(Vertical Stirrups)}$$

If the stirrup is inclined at an angle θ to the horizontal, then,

$$M_I = \frac{1.5\, a_s f_s jd}{\sin\theta}$$

And when $\theta = 45^\circ$ (7)

$$M_I = 2.1\, a_s f_s jd \quad \text{(Stirrups inclined 45°)}$$

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