5 4. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ and $\frac{h}{k}=\frac{l}{m}=\frac{n}{p}$

prove that $\frac{(a+c+e)(h+l+n)}{(b+d+f)(k+m+p)}=\frac{ah+cl+en}{bk+dm+fp}$.

5 (*a*). Reduce $\frac{ab(x^2-y^2)+xy(a^2-b^2)}{ab(x^2+y^2)+xy(a^2+b^2)}$ to its lowest terms.

8 (*b*). If $xy+yz+zx=1$ prove that

$$\frac{x}{1-x^2}+\frac{y}{1-y^2}+\frac{z}{1-z^2}=\frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}.$$

4. Prove that

5 (*a*). $\frac{2\{x+2+\sqrt{(x^2-4)}\}}{x+2-\sqrt{(x^2-4)}}=x+\sqrt{(x^2-4)}$.

5 (*b*.) $(b+c-a)a^{\frac12}+c+a-b)b^{\frac12}+(a+b-c)c^{\frac12}=$
$(a+b+c)(a^{\frac12}+b^{\frac12}+c^{\frac12})-2(a^{\frac32}+b^{\frac32}+{}^{\frac32})$.

6. Solve the equations—

5 (*a*). $(b-c)(x-a)^3+(c-a)(x-b)^3+(a-b)(x-c)^3=0$.

8 (*b*). $x+y=4xy$; $y+z=2yz$; $z+x=8zx$.

8 (*c*). $x+y+z=0$.
$ax+by+cz=0$.
$bcx+cay+abz+(a-b)(b-c)(c-a)=0$.

8 (*d*). $\frac{x-1}{x+3}+\frac{x-8}{x+1}+2=0$.

SOLUTIONS.

1. (1.) On dividing by $x+\cdot7$ (use Horner's method) we find that expression equals $(x^4+\cdot8x^3-166\cdot21x^2-49\cdot653x+115\cdot7571)(x+.7)-\cdot02997$, which, when $x=-\cdot7$, becomes $-\cdot02997$, the factor $x+\cdot7$ vanishing.

(2.) Similarly this expression becomes a^2+ag+p^2.

2. The remainder on dividing by $x+b$ is $a(-b)^2+b(-b)+c=ab^2-b^2+c$, and required condition is hence evidently $ab^2-b^2+c=o$.

(*a*). $=(a-b)\{a-2b+3(a+b)+4(a-b)\}=(a-b)(8a-8b)$.

(*b*). Putting $a=b$, we see that $a-b$ is a factor, and thence by symmetry $b-c$ and $c-a$; also x and $x-1$ are evidently factors, the expression vanishing when $x=o$ and $x=1$.

Also the terms of six dimensions $(abcx^3)$ evidently destroy each other. Hence the only literal factors are the above, and we may assume expression

$$=A(a-b)(b-c)(c-a)(x-1)x.$$

And assigning numerical values to a, b, c, x (say $a=1$, $b=2$, $c=3$, $x=2$) we see that $A=-1$. Whence expression equals $-(a-b)(b-c)(c-a)(x-1)x$.

3. $b^2=4ac$.

(*a*). Expression $=(a-b)^4-4(a-b)^2(a^2+b^2)+4(a^2+b^2)^2$.
$=\{a-b)^2-2(a^2+b^2\}^2$.

The square root of which is $\pm\{(a-b)^2-2(a^2+b^2)\}=\mp(a+b)^2$.

(*b*). $x-2$, $x-1$, x, $x+1$, will represent any four consecutive numbers. The sum of their squares is $4x^2-4x+6=(2x-1)^2+5$, and whatever be the value of x, $(2x-1)^2$ is a perfect square.

4. Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=x$; $\therefore a=bx$, etc., and $a+c+e=(b+d+f)x$

$\frac{h}{k}=$ etc., $=y$; $\therefore h=ky$, etc; and $h+l+n=(k+m+p)y$.

Hence $\frac{(a+c+e)(h+l+n)}{(b+d+f)(k+m+p)}=xy$.

Also $\frac{ah+cl+en}{bk+dm+fp}=\frac{bkxy+dmxy+fpxy}{bk+dm+fp}=xy$.

(*a*) $\frac{abx^2-aby^2+a^2xy-b^2xy}{abx^2+aby^2+a^2xy+b^2xy}=\frac{ax(bx+ay)-by(ay+bx)}{ax(bx+ay)+by(ay+bx)}$
$=\frac{ax-by}{ax+by}$.

(*b*) $\frac{x}{1-x^2}+\dots\dots=\frac{x(1-y^2)(1-z^2)+\dots\dots}{(1-x^2)(1-y^2)(1-z^2)}$

$=\frac{x+y+z-x(xy+xz)-y(yz+yx)-z(zx+zy)+xyz(xy+yz+zx}{(\quad)(\quad)(\quad)}$

$=\frac{x+y+z-x(1-yz))-y(1-zx)-z(1-xy)+xyz}{(\quad)(\quad)(\quad)}$, $\because xy+yz+zx=1$

$=\frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$.

5. (*a*) $=\frac{2(x+2+\sqrt{x^2-4})^2}{(x+2)^2-(\sqrt{x^2-4})^2}$, rationalizing the denominator.

$=\frac{2\{x^2+4x+4+2(x+2)\sqrt{x^2-4}+x^2-4\}}{x^2+4x+4-x^2+4}$

$=\frac{2\{2x(x+2)+2(x+2)\sqrt{x^2+4}\}}{4(x+2)}=x+\sqrt{x^2-4}$.

(*b*) Transfer $2(a^{\frac32}+\dots)$ to the left side, which then becomes $(b+c+a)a^{\frac12}+\dots.+\dots=(a+b+c)(a^{\frac12}+b^{\frac12}+x^{\frac12})$.

6. (*a*) $(b-c\{x^3-3ax^2+3a^2x-a^3\}+\dots+\dots=o$.

And without multiplying out it may be seen that coeffs. of x^3 and x^2 disappear, and

$$x=\frac{a^3(b-c)+\dots+\dots}{3\{a^2(b-c)+\dots+\dots\}}=\frac{(a-b)(b-c)(a-c)(a+b+c)}{3(a-b)(b-c)(a-c)}$$
$$=\tfrac13(a+b+c).$$

(*b*). Transform to $\frac{1}{y}+\frac{1}{x}=4$., &c.. finding $\frac{1}{x}$, $\frac{1}{y}$, $\frac{1}{z}$.

$x=\frac{2}{5}$, $y=\frac{2}{3}$, $z=2$.

(*c*). $x=b-c$, $y=c-a$, $z=a-b$.

(*d*). $x=-1\pm\sqrt{2}$.

EUCLID.

TIME—TWO HOURS AND A HALF.

Examiner—J. J. TILLEY.

Values.

6 1. Define Right Angle, Rectilineal Figure, Scalene Triangle, Postulate, Parallel Straight Lines, Gnomon.

10 2. (*a*) If two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of the one greater than the angle contained by the two sides of the other, the base of that which has the greater angle shall be greater than the base of the other.

4 (*b*) What restriction does Euclid make in his construction, and why?

10 3. The opposite sides and angles of a parallelogram are equal to one another, and the diameter bisects it, that is, divides it into two equal parts.

10 4. To describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

10 5. If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts is equal to the rectangle contained by the two parts together, with the square on the aforesaid part.

10 6. If a straight line be divided into two equal, and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line, and of the square on the line between the points of section.

10 7. Through a given point draw a line, so that the parts of it, intercepted between that point and perpendiculars upon it from two other given points, may be equal to each other.