11. Solution of the Gaussian. §43.

12. Analysis of solvable irreducible equations of the fifth degree. The auxiliary biquadratic either is irreducible, or has an irreducible sub-auxiliary of the second degree, or has all its roots rational. The three cases considered separately. Deduction of Abel's expression for the roots of a solvable quintic. §58-74.

PRINCIPLES.

§1. It will be understood that the surds appearing in the present paper have *prime numbers* for the denominators of their indices, unless where the contrary is expressly stated. Thus, $2^{\frac{1}{15}}$ may be regarded as $h^{\frac{1}{5}}$, a surd with the index $\frac{1}{5}$, h being $2^{\frac{1}{3}}$. It will be understood also that no surd appears in the denominator of a fraction. For instance, instead of $\frac{2}{1 + \sqrt{-3}}$ we should write $\frac{1 - \sqrt{-3}}{2}$. When a surd is spoken of as occurring in an algebraical expression, it may be present in more than one of its powers, and need not be present in the first.

§2. In such an expression as $\sqrt{2} + (1 + \sqrt{2})^{\frac{1}{3}}$, $\sqrt{2}$ is *subordinate* to the *principal* surd $(1 + \sqrt{2})^{\frac{1}{3}}$, the latter being the only principal surd in the expression.

§3. A surd that has no other surd subordinate to it may be said to be *of the first rank;* and the surd $h^{\frac{1}{c}}$, where h involves a surd of the $(a - 1)^{\text{th}}$ rank, but none of a higher rank, may be said to be *of the* a^{th} *rank.* In estimating the rank of a surd, the denominators of the indices of the surds concerned are always supposed to be prime numbers. Thus, $3^{\frac{1}{6}}$ is a surd of the second rank.

§4. An algebraical expression in which $\Delta_1^{\frac{1}{m}}$ is a principal (see §2) surd may be arranged according to the powers of $\Delta_1^{\frac{1}{m}}$ lower than the m^{th}, thus,

$$\frac{1}{m}\left(g_1 + k_1 \Delta_1^{\frac{1}{m}} + a_1 \Delta_1^{\frac{2}{m}} + b_1 \Delta_1^{\frac{3}{m}} + \ldots + e_1 \Delta_1^{\frac{m-2}{m}} + h_1 \Delta_1^{\frac{m-1}{m}}\right) \quad (1)$$

where g_1, k_1, a_1, etc., are clear of $\Delta_1^{\frac{1}{m}}$.