SECTION C.—ALGEBRA.

13. Explain why the product is a^{12} when a^5 is multiplied by a^7, and why the quotient is a^5 when a^8 is divided by a^3.

14. Obtain $(x^2+3x+15)^2-(x^2-3x+15)^2$ in its simplest form, and find its value when $2x=-5$.

15. Simplify the expressions :—

$$(a)\ \frac{x-a}{\frac{1}{a}-\frac{1}{b}}\times\frac{a-b}{1-\frac{a}{x}}$$

$$(b)\ \left(\frac{1}{2}+\frac{1}{3x}\right)\div\left(9x-\frac{4}{x}\right).$$

16. Find the greatest common measure of $x^4-5x^3-6x^2+36x-7$ and $3x^3-13x^2+43x-8$, and write down these expressions in factors.

17. Solve the equations :—

$$(a)\ x-\frac{2x-0{\cdot}3}{0{\cdot}7}=\frac{5-x}{0{\cdot}35}.$$

$$(b)\ \frac{3y}{4}-\frac{2x}{5}=\frac{13}{5},\ \frac{x}{4}+\frac{y}{5}=\frac{19}{12}.$$

18. A sum of £23 14*s.* is to be divided between A, B, and C; if B gets 20 per cent. more than A and 25 per cent. more than C, how much does each get?

SECOND STAGE.

(Three Hours are allowed for this Paper.)

SECTION A.—ARITHMETIC AND ALGEBRA.

21. Find the value of

$$\left(x+\frac{a}{b}\right)\left(x+\frac{b}{a}\right)-\left(x-\frac{a}{b}\right)\left(x-\frac{b}{a}\right)$$

when $x=\frac{1}{a^2+b^2}$.

22. Reduce the following expression to its simplest form :—

$$\left(\frac{a-b}{a+b}\right)-\left(\frac{a}{a-b}+\frac{b}{b-a}\right)^2$$

and find its value expressed as a decimal when

$$a=2 \text{ and } b=\sqrt{5}=2{\cdot}23607.$$

23. Find all the values of x or of x and y which satisfy the following equations :—

$$(a)\ \frac{9}{x}+\frac{25x}{x-1}+9=9.$$

$$(b)\ (x^2-4x+3)^2-8(x^2-4x+3)+0.$$

$$(c)\ 4(x^2-y^2)=35,\ x-2y=2.$$

24. The first of two pictures is 1 ft. 6 in. by 2 ft., the second 2 ft by 2 ft. 6 in.; they are to be framed in the same way; if the glass and frame of the former cost 7*s.* 6*d.* and that of the latter 11*s.* 2*d.*, what is the price of the glass per square foot, and of the frame per foot of length?

25. Find the first five terms of the square root of $1+x$, and by means of them show that $\sqrt{(101)}=10{\cdot}0498756$.

26. A racecourse is 3,000 ft. long; A gives B a start of 50 ft. (so that B has to run 2,950 ft.), and loses the race by a certain number of seconds; if the course had been 6,000 ft. long, and they had both kept up the same speed as in the actual race, A would have won by the same number of seconds. Compare A's speed with B's.

SECTION B.—GEOMETRY.

27. If a straight line be bisected and produced to any point, the rectangle contained by the whole line thus produced and the part produced, together with the square on half the line bisected, is equal to the square on the straight line made up of the half and the part produced.

In the triangle ABC let AB equal AC, produce BC to D, join AD; show that the square on AD exceeds the square on AB by the rectangle BD, DC.

28. If an angle at the centre of a circle, and another at the circumference stand on the same part of the circumference, show that the former angle is double the latter. Only one case need be proved.

Draw a straight line which shall divide a given circle into two segments, such that the angle in the one shall be three times the angle in the other.

29. Show that the angle in a semicircle is a right angle.

If the diagonals of a quadrilateral are equal, and bisect each other, show that the quadrilateral is a rectangle.

30. Construct a square equal to one-third of a given square.

31. Describe three circles of given radii such that two such shall touch each other externally and the third internally. What relation must exist between the radii if the construction is to be possible?

32. Let PA and PD be two lines of given length inclined at any angle; in PA take any point B; find a point C in PD (or PD produced) such that the rectangle AP, PB shall equal the rectangle CP, PD. How could you tell by merely considering the angles PAD and PDB, whether C falls in PD, at D, or in PD produced?

THIRD STAGE.

SECTION A.—MISCELLANEOUS.

41. Find all the values of x and y that satisfy the equations

$$\frac{x-y+1}{x+y+2}+\frac{x-y+1}{x+y}=4.$$

$$9+\frac{16}{x-y+1}+\frac{4}{x+y}=0.$$

42. Show how to find the number of homogeneous products of r dimensions, which can be formed of powers of n letters $a, b, c, \ldots$

How many sets of positive integral values of u, x, y, and z, satisfy the equation

$$u+x+y+z=12.$$

N.B.—Zero (0) is to be reckoned a positive integral value of u, x, y, or z.

43. If a and b are any two numbers, and A, G, H three other numbers, such that a, b, A are in arithmetical progression, a, b, G in geometrical progression, and a, b, H in harmonical progression, show that

$$4H(A-G)(G-H)=G(A-H)^2.$$

44. If two parallelograms have a common diagonal, show that the straight lines joining the angular points on one side of the diagonal is parallel to the straight line joining the angular points on the other side of the diagonal.

45. Let two circles touch internally at A, and let the radius of the one be equal to the diameter of the other circle; draw AB the diameter of the larger circle passing through A, and BP to touch the smaller circle in P; join AP; show that the square on BP is three times the square on AP.

46. Show how to construct a rhombus, having given its angles, and the radius of its inscribed circle.

HONORS EXAMINATION.

You may not answer more than ten questions.
The value attached to each question is 50.
Three hours are allowed for this paper.

61. If x', x'', y', y'' are the values of x and y, which satisfy the equations

$$ax+by+c=0.$$
$$lx^2+my^2+n=0.$$

Show that $a^2(m+n)+b^2(n+l)+c^2(l+m)=0$ provided

$$x'x''+y'y''+1=0.$$

62. There are n things, p of one sort, q of another sort, r of a third sort, and so on; find the number of combinations that can be formed of them, taken m at a time.

Two counters are marked A, three B, five C, and two D; how many different combinations can be formed of the counters taken five at a time.

63. When m and n are small compared with x show that—

$$\log(x-m)-\log x=\frac{m}{n}\left(1+\frac{n-m}{2x}\right)(\log(x+n)-\log x) \text{ approxi'ly.}$$

Given $\log 1{\cdot}01=0{\cdot}0043214$, calculate $\log 1{\cdot}0075$.

64. Sum the following series :—

$$(a)\ \frac{1}{p(p+1)}+\frac{1}{(p+1)(p+2)}+\frac{1}{(p+2)(p+3)}+\ldots\text{ad. inf.}$$

$$(b)\ 3\times5+5\times7+7\times9+\ldots+(2n+1)(2n+3).$$

$$(c)\ 3m+5(m-1)+7(m-2)+\ldots+(3+2(m-1))1.$$

65. $ABCD$ is a quadrilateral, having the sides AB and DC parallel to each other, and together equal to the side BC; show that the straight lines, which bisect the angles B and C, intersect in AD.