which $x=1$. Let r be radius of the circle; x, y sides of rectangle; then we are to make $\pi r^2 - xy$ a minimum where $x^2+y^2=4r^2$. Substitute for y and proceed as before, when $x=y$, or greatest rectangle is a square.

3. In the solution of $ax^2+bx+c=0$, interpret the results when

$$(1)\ a=b=0; \qquad (2)\ b^2 \lesseqgtr 4ac.$$

(1) Roots are ∞, ∞. (Todhunter's Larger Algebra, secs. 342 and 343.)

(2) Roots are real and unequal, real and equal, or imaginary.

4. If α, β be the roots of $x^2+px+q=0$, and α', β' those of $x^2+px+\frac{1}{9}(2p^2+q)=0$, then $\alpha, \alpha', \beta', \beta$ form an Arithmetic series.

$$\alpha=\frac{\sqrt{p^2-4q}-p}{2}, \quad \beta=\text{etc.}$$

$$\alpha'=\frac{\sqrt{p^2-4q}-3p}{6}, \quad \beta'=\text{etc.}$$

We have $\alpha+\beta'=2\alpha'$, $\alpha'+\beta=2\beta'$; $\therefore$ etc.

5. Determine the conditions that ax^4+bx^2+c and cx^4+bx^2+a may have a common divisor of the form x^2+px+q.

Divide ax^4+bx^2+c by x^2+px+q, and put remainder $x^2(ap^2-aq+b)+apqx+c=0$; similarly with cx^4+bx^2+a we find

$$x^2(cp^2-cq+b)+cpqx+a=0.$$

Eliminating in turn first and last terms of these expressions we find two values for x, equating which gives condition

$$\left\{(p^2-q)(a+c)+b\right\}^2+p^2q^2b(a+c)=0.$$

6. When is one quantity said to vary as another?

If $x \propto y \propto z$, show that constants k, l, m exist such that

$$l(x-ky)=m(y-lz)=k(z-mx).$$

Bookwork.

Solving between x and y, $x=k'y$ where k' is a constant, and so on; $\therefore x \propto y$ by definition, and so for z.

7. The sum of n terms of a certain series is $\frac{1}{3}n(n+1)(n+2)$; shew that the sum of the differences between the 1st and 2nd, 2nd and 3rd, ...$(n-1)^{\text{th}}$ and n^{th} terms is

$$(n-1)(n+2).$$

The n^{th} term of above series $=S_n-S_{n-1}$

$$=\frac{n(n+1)(n+2)}{3}-\frac{(n-1)n(n+1)}{3}=n^2+n;$$

$\therefore$ sum is required of

$$\left\{(2^2+2)-(1^2+1)\right\}+\dots$$
$$+\left\{(n^2+n)-\left((n-1)^2+n-1\right)\right\}$$
$$=2(2+3+\dots+n)=(n-1)(n+2).$$

8. Find the sum to n terms of a Geometric series, having given the first term and common ratio.

If between each pair of the quantities x, x^2; x, x^3; x, x^4; ....n Geometric means be inserted, and r_1, r_2.... be the common ratios, then

$$\frac{r_2}{r_1}+\frac{r_3}{r_2}+\dots+\frac{r_{n+1}}{r_n}=n\,.\,x^{\frac{1}{n+1}}.$$

Bookwork.

For the first series,

$$xr_1^{n+1}=x^2, \ \therefore r_1=x^{\frac{1}{n+1}};$$

for the second

$$xr_2^{n+1}=x^3, \ \therefore r_2=x^{\frac{2}{n+1}}, \text{ etc.};$$

$$\therefore \frac{r_2}{r_1}=x^{\frac{1}{n+1}}, \text{ and sum}=n\,.\,x^{\frac{1}{n+1}}.$$

9. In forming the combination of n things r together, find for what value of r the number of combinations is greatest.

A committee of 8 is to be selected by taking a certain number (a) from a party of 13, and the remainder from a party of 8. What is the value of a that the selections may be made in the greatest number of ways; and how often will A of the first party and B of the second party find themselves in company?

Bookwork.

The value of a for which the number of combinations is greatest is 6 or 7. When $a=6$, 5 other men may be chosen from the first party in $\frac{\lfloor 12}{\lfloor 5 \lfloor 7}$ put A in each of these, one can be selected from the second party in 8 ways; put B along with each of them, so that on the whole when $a=6$, A and B will be together in $\frac{\lfloor 12}{\lfloor 5 \lfloor 7}\times 8$ ways. Similarly when $a=7$.