7. What value of $x$ will make $x^3+3cx^2+2c^2x+5c^3$ equal to the cube of $x+c$?

$(x+c)^3=x^3+3cx^2+3c^2x+c^3=x^3+3cx^2+2c^2x+5c^3$; $\therefore x=4c$.

8. If $a^2+b_2=c^2$ and $s=a+b+c$, prove that $(2s-a)^2+(2s-b)^2=(2s-c)^2$.

This question is incorrect.

9. Having 75 minutes at my disposal, how far can I go in a carriage at $6\frac{2}{3}$ miles an hour, having to walk back at $3\frac{3}{4}$ miles an hour?

Let $x$=distance in miles.

$$\frac{x}{6\frac{2}{3}}+\frac{x}{3\frac{3}{4}}=\frac{75}{60} \ : x=3.$$

10. I row $a$ miles down a stream in $b$ minutes and return in $c$ minutes; find the rate at which I row in still water, and the rate at which the stream flows.

Let $x$=man's rate in still water, $y$=stream's rate per hour. Man goes down at $x+y$ rate, and up at $x-y$ rate; hence

$$\frac{a}{x+y}=\frac{b}{60},\quad \frac{a}{x-y}=\frac{c}{60},\quad x=\frac{b+c}{c-b}y,$$

$$x=\frac{30a(b+c)}{bc},\quad y=\frac{30a(c-b)}{bc}.$$

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ALGEBRA (SECOND CLASS.)

1. Find the value of $x^5+x^4-166x^3-166x^2+81x+81$ when $x=-\cdot7$; and the value of $x^3-3px^2+(3p^2+q)x-pq$ when $x=a+p$. (Arrange the latter result according to powers of $a$.)

The result can be obtained by substitution or by Horner's Method of Division.

The second part may be worked as under: In the expression put for $x$ its value, thus,

$(a+p)x^2-3px^2+(3p^2+q)x-pq$
$=(a-2p)x^2+(3p^2+q)x-pq$;

again, put for $x$ its value and reduce,

$(a^2-ap+p^2+q)x-pq$,

and so on, the result being $a^3+aq+p^3$.

2. What is the condition that $x+b$ shall be a factor of $ax^2+bx+c$?

Find the factors of

(a.) $(a^2-ab)+2(b^2-ab)+3(a^2-b^2)+4(a-b)^2$; and (b). $(ax+b)(bx+c)(cx+a)-(ax+c)(bx+a)(cx+b)$.

Put $x+b=0$, we have $ab^2-b^2+c=0$ for the condition.

(a) $(a-b)(8a-3b)$; (b) $x(1-x)(b-c)(c-a)(a-b)$.

3. What must be the relation among $a$, $b$, $c$ that $ax^2+bx+c$ may be a perfect square?

(a.) Extract the square root of $(a-b)^4-4(a^2+b^2)(a-b)^2+4(a^4+b^4)+8a^2b^2$.

(b.) If 5 be subtracted from the sum of the squares of any four consecutive numbers the remainder will be a perfect square. (Prove this.)

Put the expression=0 and solve as a quadratic equation. Then the condition required is that for equal roots, viz.: $b^2=4ac$.

(a) $(a-b)^2-2(a^2+b^2)=-(a+b)^2$.

(b) Take for the consecutive integers $x-1$, $x$, $x+1$, $x+2$; we have

$(x-1)^2,\ +x^2,\ +(x+1)^2,\ +(x+2)^2-5$
$=4x^2+4x+1=(2x+1)^2$.

4. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ and $\frac{h}{k}=\frac{l}{m}=\frac{n}{p}$

prove that

$$\frac{(a+c+e)(h+l+n)}{(b+d+f)(k+m+p)}=\frac{ah+cl+en}{bk+dm+fp}.$$

(a.) Reduce $\frac{ab(x^2-y^2)+xy(a^2-b^2)}{ab(x^2+y^2)+xy(a^2+b^2)}$ to its lowest terms.

(b.) If $xy+yz+zx=1$ prove tha

$$\frac{x}{1-x^2}+\frac{y}{1-y^2}+\frac{z}{1-z^2}=\frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}.$$

Let $\frac{a}{b}=x$, &c., $\therefore a=bx$, &c.;

$\frac{h}{k}=y$, &c., $\therefore h=ky$, &c.;

hence left-hand members$=xy$.

Also we have $ah=bkxy$, &c.=&c.;

$\therefore$ right-hand member$=xy$.

(a) $\frac{ab(x^2-y^2)+xy(a^2-b^2)}{ab(x^2+y^2)+xy(a^2+b^2)}$
$=\frac{(bx+ay)(ax-by)}{(bx+ay)(ax+by)}=\frac{ax-by}{ax+by}$.

(b) By simplifying the left-hand member of the equality the numerator is

$x+y+z-z^2(x+y)-y^2(x+z)-x^2(y+z)$
$+xyz(xy+xz+yz)$.

From the given equality

$z^2(x+y)=z-xyz$, $y^2(x+z)=y-xyz$, &c.;

$\therefore$ the numerator $=$

$x+y+z-x+y-z+4xyz=4xyz$,

which is the numerator of the right-hand member; and the denominators are the same.