but each sound is the imitation without the charm and music of song. This kind of reading and speaking often marks the utterance of clergymen, and hence receives the appropriate name of "pulpit tones." We cannot read or speak naturally and with right expression without inflection. Every word and every syllable we utter must be inflected, and he who cannot inflect naturally as he reads has yet to learn the art of reading.

ELEMENTARY EXERCISES.

I. Sound the following vowels each in one breath prolonged until "out of breath." There must be no break, no jerking in the sound; it must be regular and pure. The pitch at first should be that about half way between the lowest and highest notes of a natural octave, varying, however, from *mi* to *la*. In more matured voices than those of children the bass voice may be practiced on the notes D E F G and the tenor on G A B C.

(1) Sound a——a as in ah, continuously, then, in succession, o——o, e——e, ā——ā, awe——awe, oo——oo, oi——oi, ou——ou, l——l, m——m, n——n. The two letters before and after each dash do not signify repetition but continuation.

In this and all similar exercises, inhalation must be regular and according to the rule given for abdominal breathing. As the sound is poured out the abdomen is drawn inwards, the diaphragm by that action is raised and the lungs are slowly emptied. When thus producing sound the learner or teacher should keep such control over the tone as to sustain its purity from roughness to the end. The action of breathing is centred in the waist muscles and the abdomen, not in the throat, as in coughing or gaping.

II. *The Crescendo Exercise.*—Use the vowels in Exercise I. in the following manner: Begin *ah* with a gentle effort, producing a pure but *piano* sound, and increase its force as it advances until it reaches a full *forte*. The sound may be compared to the thin edge of the wedge advancing until it reaches the broad end.

III. *Diminuendo Exercise.*—Reverse the above exercise. Commence with full force of the voice and gradually diminish that force until it softens to the mildest piano. Use all the vowels of Exercise I.

IV. The combination of the increasing and diminishing practice produces those tones so appropriate to reverential and solemn thoughts and language. The commencing and ending of each sound are softer and the fullest force is heard in the middle of the tone, but the changes must not be jerky, but uniformly swelling to that centre and then diminishing.

Exercises for Expulsive and Explosive Forces.—These exercises demand greater muscular effort, and should not be given to very young children nor practised too long or too often by older pupils and learners. But they are indispensable for all powerful energetic utterances, and, judiciously used, not only strengthen the vocal muscles, but carry a stirring, commanding and irresistible force with them. In the practice on the vowels of Ex. I. the preparation is similar. The inhalation commences and is full. A momentary pause prepares for the effort, as if for a leap. The glottis is closed, the cords rigid and in contact, and the breath held. Then the action: the voice is sent forth with clearness, prompt and with great force, but slightly diminishing that force to the end. It is the broad end of the wedge narrowing to the thinner end, but throughout sustaining the purity of tone. This describes the *expulsive force.*

The Explosive Force is produced similarly, but the force is sustained to the end, and the shout is brief and the stoppage prompt.

IV. The *Tremor* of the voice. This is one of the most necessary functions of the voice. It is never omitted in natural expression, when the heart is deeply moved, but rarely produced by the reader, and this omission is one of the evidences that a knowledge of the subject matter is enough for expression, as some teachers of a certain school maintain. It must be an acquired and a voluntary act in harmony with the feeling. It is not the "shake" of the singer, for that is a change of pitch. Dr. Rush compares it to the "gurgle" in the throat, and in the exercise for its production it has the sound of the gurgle. It is indispensable to all expressions of strong emotion.

In the next article additional explanations of the subject will be given, with illustrative examples for practice.

Mathematics.

All communications intended for this department should be sent before the 20th of each month to C. Clarkson, B.A., Seaforth, Ont.

CONSTRUCTIVE GEOMETRY.

1. AB being a given lignet to construct the line AB $\sqrt{2}$

Draw BC perp. to AB at its extremity, then AC is = AB $\sqrt{2}$.

For, $AC^2 = AB^2 + BC^2 = 2AB^2$, $\therefore$ AC = AB $\sqrt{2}$

Cor.—The sq. on the diagonal is twice the given square.

2. To construct the line AB $\sqrt{3}$. (This figure and all following are easily drawn.)

Produce AB until it is=2AB, on this line AC describe an equilateral triangle ADC, join BD, then BD=AB $\sqrt{3}$, for $AD^2 = 4AB^2$;

but $AD^2 - AB^2 = BD^2 = 3AB^2$; thus BD=AB $\sqrt{3}$.

3. To construct AB $\sqrt{5}$.

From B draw BC perp. to AB and=2AB, then AC = AB $\sqrt{5}$.

For $BC^2 = 4AB^2$, $\therefore$ $AC^2 = 5AB^2$, thus AC= AB $\sqrt{5}$.

4. To construct AB $\sqrt{7}$.

From B draw BD perp. to AB and=AB $\sqrt{3}$ by No. 2.

Also produce BA to E till BE=2BA, then ED= AB $\sqrt{7}$.

5. To find a point C in a given line AB, such that AC^2=AB. BC. (Euc. II. 11.)

Considering AC^2= AB. BC as an algebraic equation and solving as a quadratic we get AC=$\frac{1}{2}$ (AB $\sqrt{5}$−AB), and this represents the line to be constructed. Construct AD=AB $\sqrt{5}$ as in No. 3; take DF=AB and bisect AF in G, and take AC=AG. Then C is evidently the point required in B.

As the $\sqrt{5}$ has two signs, if we take the negative sign, the algebraical expression assumes the form AC= −(AB $\sqrt{5}$+AB). Now the negative sign is interpreted by a line drawn in the opposite direction, hence take DF_1=AB in AD *produced*, bisect AF_1 in G_1 and *produce* BA to C_1 so that $AC_1 = AG_1$, then C_1 is the point required in BA produced. This construction requires no proof other than the equation it represents. The proof, however, is easy, for $AD^2 = 5AB^2$, and $=(AF+FD)^2$ $=(2AC+AB)^2$

or $5AB^2 = 4AC^2 + AB^2 + 4ACAB$, *i.e.*, $AC^2 = AB^2 - ACAB$,

or AC^2=AB(AB−AC)=AB, BC.

6. Find the side of the square which is equal to a given rectangle. (Euc. II. 14.)

Let AB be one side of the rectangle; produce AB to C so that BC is equal to the other side of the rectangle, $\therefore$ AC=x+y if we let x and y denote the sides of the rectangle. Bisect AC in D, then CD=$\frac{1}{2}$(x+y) and BD=$\frac{1}{2}$(x+y)−y=$\frac{1}{2}$(x−y)

Now $\left(\frac{x+y}{2}\right)^2 - \left(\frac{x-y}{2}\right)^2 = \frac{1}{4}(x^2+y^2+2xy) - \frac{1}{4}(x^2+y^2-2xy) = xy$, the given rect. That is $CD^2 - BD^2$ = the given rectangle. We have therefore to construct a right-angled triangle with CD for hypot. and BD for base. From B draw any perp., and from centre D with distance C, describe an arc cutting this perp. in E; then BE is evidently the side of the sq. required.

7. Divide a given line AB in the point C so that the rectangle AC, CB may be equal to the difference between the squares on AC and CB.

Let AB=a, and BC=x, $\therefore$ BC=a−x. Then we require to take C so that $x^2-(a-x)^2 = x(a-x)$; or $x^2+ax-a^2=0$; that is $x^2=a(a-x)$. The problem is thus seen to be identical with No. 5, and the same construction will apply. For variety, however, we will put the two roots in slightly different forms, and thus vary the construction. We can say that the roots of $x^2+ax-a^2=0$ are

$x_1 = \sqrt{\left[\left(\frac{a}{2}\right)^2 + a^2\right]} - \frac{a}{2}$; and

$-x_2 = \sqrt{\quad}$ " " " $] + \frac{a}{2}$. Then draw BD perp. to AB and=$\frac{1}{2}$ a. Join AD, and from centre D and with DB for radius describe a circle cutting AD in E and AD produced in F. Then $x_1 = AE_1$ and x_2=AF, as is manifest from the figure.

NOTE.—These examples are sufficient to show some of the advantages of the algebraical method of solving problems in geometry. The algebraical analysis gives a general expression which includes all possible geometrical constructions; and a mere examination of the form of this expression will usually settle the limits of the possible solutions. By introducing the *Principle of Descartes, i.e.*, the conception of negative lines, this method enables us to group together related problems and bring them under one generalised form. If the root of the algebraic equation assumes an imaginary form the problem is impossible in the ordinary acceptation of the term. But imaginary algebraic quantities are capable of a geometrical interpretation, and we may return to this point at some other time. The matter of this paper is drawn chiefly from Prof. Dupuis' *Elementary Synthetic Geometry*, to which we have previously directed attention.

PROBLEMS SENT FOR SOLUTION.

58. SUPPOSE 5 candidates are examined for 2 scholarships, and that A obtains $\frac{2}{9}$ of the whole number of marks given; B twice as many as A gets more than C, who obtains 3 times as many as B gets more than D; that D obtains $\frac{1}{2}$ as many as A, B, C together, and E $\frac{1}{3}$ more than the excess of the sum of A, B and C's marks together over D's. Determine the successful candidate. (Sent by H.J.S., King.)

59. A market-woman who has an exact number of dozens of eggs finds that if she counts them by 8, or by 10, or by 20, there are always 4 eggs left. What is the least number of dozens she can have?

60. On counting out the marbles in a bag by 20 at a time, or by 24, or by 30, there are always 15 marbles left. What is the least number of marbles there can be in the bag? (Sent by N.S.E., Rockford.)

CORRESPONDENCE.

DEAR SIR,—I wish to call your attention to the solution of No. 70 of last year, given in March number of EDUCATIONAL JOURNAL.

It appears to me that the solver has left out of consideration the fact that the rope will overlap on the circumference of the given circle if the animal is to graze on the *outside* of it, and thus a curve will be formed that is not an arc of a circle.

But the solver has given a neat solution to the problem I proposed in the JOURNAL about three years ago, namely to find the length of the rope when the horse is fastened to a tether on the *inside* of the given acre.

My solution of No. 70 is as follows :—

Let v=radius of given circle, and x the required rope,

Then $\int \frac{x^2.dx}{2v} = \frac{x^3}{2.3v}$ = area of half surface formed by overlapping, and area of semicircle $= \frac{\pi x^2}{2} \therefore \frac{x^3}{3v} + \frac{\pi x^2}{2} = 160$

whence x is 49,318 yards. Yours truly,

WM. W. IRELAND.

Madoc, Ont.

QUESTIONS FOR THE EVENING HOUR.

1. HAVE I *taught* to-day?
2. Am I making my pupils self-helpful?
3. Have I, in this day's work comprehended the true end of education?
4. Are my pupils learning self-control from their association with their teacher?
5. Have I, to-day, by word and action, taught honesty, integrity and truthfulness?
6. Am I developing in my pupils the power of organized and independent thought?
7. How would I like to go to school to such a teacher as I am?
8. Are my pupils better children in their homes because of my influence?
8. Do I treat my pupils as I would like to have a teacher treat my child?
10. How often do I see school work as it appears to parents?—*Supt. C. H. Gurney.*