statements are necessary to define a square. Again, to describe a square as 'a quadrilateral rectilinear figure whose sides are equal and its angles right angles,' is to err slightly in excess; for if any two adjacent sides be equal and its angles right angles, or its sides be equal and *one* angle be a right angle, our definition will be suited by the square and by no other figure. And this is just what a definition should do—it should describe the object or operation referred to, and no other, and should only state what is entirely necessary. As an instance of inaccurate definition of an *operation*, suppose we intend to describe upon a given line a triangle equal in all respects to another, it would be saying too little if we said that two sides of the one were to be equal to two sides of the other, each to each, for it is also necessary that the angle between them should be equal, or that the third side should be equal. Similarly, we should be erring in excess if we said that two angles and two sides should be equal, since it is ample to say that two angles and one side should be so. Similarly, an equilateral pentagon is not necessarily regular, nor yet is an equiangular pentagon so; only a pentagon at once equilateral and equiangular answers the definition of a regular pentagon.

Having thus noticed what are the requirements of an accurate definition, it is necessary both to use carefully our words when defining any figure or operation, and also to examine carefully the terms in which any problem or theorem is presented to us, making sure that in our treatment of the question we really use up all the facts stated, or otherwise the proposition that we are attempting may not be true or possible, in consequence of our having omitted some material consideration. For every proposition contains certain *data*, or things granted and certain *quæsita*, or things required to be constructed or proved. Now, unless we comprehend accurately both of these, there can be little hope of a successful result. For instance, in Book I, Prop. 47, the data are that the triangle is right angled, and that the figures described on its sides are squares, and therefore have their sides equal, and their angles, as we have learned, all right angles; each of these three facts is material to the proof, and if one of them had been overlooked the proof would have failed. Similarly, the thing required to be proved is that the square on the hypothenuse is equal to the sum of the other two; any inaccuracy in stating this would really alter the object of the whole proposition. A very common instance of false definition, or of false comprehension of a statement, is to fancy that because in two triangles two sides of the one are equal to two sides of the other, each to each, and that one angle of the one is equal to one angle of the other, that therefore the triangles are equal, which is not true except when the equal angles are those between the pairs of equal sides, as in Book I, Prop. 4.

This brings us to a consideration of the next point of vital importance, and that is *accurate reasoning*. This is generally regarded as the object of *logic*, which names and explains the various errors to which argument is liable, and shows how to avoid them; but in mathematical reasoning the subject is so clear, and the terms so accurately defined, that no careful and intelligent student should ever be in doubt as to whether a piece of reasoning is sound or unsound. For example, in the proof of a proposition, he should ask at each step. Why is this true? If the answer be 'by hypothesis,' he should examine the hypothesis and see if the given statement be really there; if the answer be 'by construction,' he should examine the construction to see if he have really so constructed the figure; if the answer be that the step depends upon any definition, axiom, or previous proposition, he should see that it really is not merely dependent on, but a necessary result of that axiom or proposition. A very little practice and careful attention will at once enable any one to detect a fallacy in geometrical reasoning.—'*Euclid*,' *in Stewart's Mathematical Series.*