opening the valve the opposition to gravity decreases and movement results. This commences at once, and the velocity at a point behind the valve will be in proportion to the fall in pressure at that point. Whether a gauge located there will correctly indicate the fall in pressure will depend upon its accuracy and sensitiveness and upon the manner in which it is attached to the pipe. In this case the energy which must be put into the water to increase its velocity comes from the water itself and from the pipe walls, which are now under less stress, and which have given up a portion of the energy stored in them. This involves a change from potential to kinetic energy.

On the other hand, when the valve is moved in a closing direction, the opposition to gravity is increased, the velocity decreases and the pressure increases; the energy given up by the water as a moving body is stored in the water and pipe walls, the one being compressed and the other distended.

In both instances an unstable condition is set up, as in the first case the pressure is much below that due to the head, and in the latter much higher. The former persists until the energy taken can be restored, which occurs on the return of the wave from the reservoir, and the latter until the excess stored can be dissipated. Equal velocity changes are accompanied by equal pressure changes whether the water be accelerated or retarded.

The changes which commence at the valve are followed by changes along the pipe-line, pressure waves traversing the pipe at velocities dependent upon the material of the pipe walls and the ratio of wall thickness to diameter.

In a previous article formulæ and diagrams were given to determine the velocity of wave propagation and the maximum ram pressure attained under any given set of conditions, and reference is made thereto for these particulars.

It is to be noted that if, in a given case, a decrease in velocity of flow of 1 ft. per second is accompanied by a ram pressure of 50 lb., a like increase in velocity will be accompanied by a fall in pressure of 50 pounds.

The formula commonly used for calculating the inertia effects of velocity changes is

$$Ha = \frac{Lv}{gt} \text{ in which}$$

Ha = accelerating or retarding head in feet.
L = length of pipe in feet.
v = velocity change, feet per second.
t = time of change, in seconds.
g = 32.2 feet per second.

This considers that the pipe is rigid and the water incompressible. It necessarily includes, as a condition, that, in a uniform pipe, all the water shall have the same velocity at the same time; that any change in velocity at the lower end shall be accompanied simultaneously by the same changes throughout. This is indicated by the inclusion of L, the length of the pipe. As a matter of fact such a condition has been shown, theoretically and experimentally, to be impossible.

The most extensive and thorough experiments along these lines were made in 1897 and 1898, in Moscow, Russia, by three engineers, working under the directions of N. Simin, manager of the city waterworks. The results were worked up by N. Joukowsky, in 1898 and published in 1900.

These show conclusively that changes initiated at the valve produce changes along the pipe after a time interval dependent upon the distance and the velocity of wave propagation, and they prove, in a very satisfactory manner, the formulæ which have been developed by theoretical means.

To bring out clearly the difference in results between the correct elastic theory and the incorrect rigid formula, the following table has been prepared. It applies to a steel pipe, 24 in. diameter, ¼-in. wall, and having lengths as listed. The velocity of wave propagation is 3,370 ft. per second.

Pipe = 1,000 ft. long.

Time of gate closing, sec...	0.00	0.02	0.04	0.10	0.30	0.45	0.60
Pressure, rigid formula.....	00	675	337	135	45	30	22
Pounds, elastic formula	45	45	45	45	45	45	45

Pipe = 10,000 feet long.

Time of gate closing, seconds ..	0.00	0.02	0.10	0.30	1.00	3.00	4.00	5.00	6.00
Pressure, rigid formula	00	6750	1350	450	135	45	34	27	22
Lb., elastic formula	45	45	45	45	45	45	45	45	45

There are several interesting facts brought out in the table. In the rigid formula the pipe length has a decided influence on the calculated result, while in the elastic, it has none whatever as far as the amount of pressure is concerned, but it does control the duration of this pressure. This has been amply proved by experiment. In the rigid formula the time of gate movement has an inverse proportionate effect upon the calculated pressure, while in the elastic, within certain limits, it has no effect. There is this fact to be noted in this connection, that the rigid formula requires that the pressure shall instantly rise to the figure given and remain there during the entire closing movement of the gate, and that the elastic indicates that the rise will be gradual and in proportion to the decrease of velocity. Experiments have shown that the former is not true and that the latter is true.

The pressures agree for a closing time equal to the time required for the wave to reach the reservoir, but with the non-elastic formula, this pressure must be considered to have existed during the entire time, while in the elastic, the maximum is reached only at the end of the period. To obtain this pressure throughout the entire period, using the elastic formulæ, the gate would have to be closed instantly. With less closing time than that mentioned above the non-elastic formula gives results which are excessive, and with greater results which are too low.

There is nothing in the non-elastic formula to account for that lagging effect which is so well known and which causes recurring waves. These are perfectly accounted for by the elastic theory, and, under proper conditions, can be predetermined.

At the limits given in the table the pressures given by the elastic formulæ commence to fall off, decreasing with increase of closing time. It is readily seen that with long conduits a great variance in results will be found, and as the elastic formulæ are based on actual conditions fixed by natural laws, a choice should not be difficult to make.

The table is based on a velocity of only 1 ft. per second. For changes of 5 to 10 ft. per second the differences would be much increased, as the ram pressures in both cases increase in proportion to the velocity change.

It is evident that the non-elastic formula is based on certain assumptions, namely, that water is incompressible and that the pipe walls are rigid, which are not true; therefore the results obtained by its use cannot be correct. How great the variation will be will depend, as shown in the table, upon the time taken for the velocity change.