$$y = a\,\frac{bc + e - cd}{ad - bc}$$

UNIVERSITY PASS ALGEBRA, FIRST YEAR, 1862.

1. Explain the terms "abstract" and "concrete," and state the conditions in arithmetical algebra that one quantity may be capable of being added to, subtracted from, multiplied by, or divided by another.

If algebraic symbols denote concrete quantities which may be said to have *direction* as well as *magnitude* and to be measured from a fixed starting point, shew that "negative quantities" will receive an interpretation.

In all other cases how do we deal with such quantities?

Apply Horner's method of division to find (1) the greatest common measure of

$$2x^5 + 2x^4 - 5x^3 + 4x^2 - 9$$
$$3x^4 + 3x^3 - 10x^2 - x + 3.$$

(2) the expansion to 5 terms in ascending powers of x, of

$$\frac{4 - 11x}{1 - 5x + 6x^2}$$

(3) the remainder after dividing

$$x^n + ax^{n-1} + bx^{n-2} + \ldots + p$$

by $x - a$

Also in (2) shew by the method of induction that the co-efficient of x^n is $3^n + 3 \cdot 2^n$.

3. Investigate the rule for finding the least common multiple of two or more algebraic quantities and find those of

(1) $apx^2 + (aq + bp)x + bq$
$aqx^2 - (ap - bq)x - bp$
(2) $x^2 - xy$, $x^{\frac{1}{2}} - y^{\frac{1}{2}}$ and $xy + y^2$

and write down the greatest common measure and least common multiple of

$$ab^p c^{q+1}, \; a^{r+1} b^{p-1} c^q, \; a^3 b^{p-2} c^{2q}$$

where a, b, c are prime to one another and p, q, r are whole numbers.

4. If m and n denote abstract whole numbers, shew that $m \times n = n \times m$.

Assuming this law to hold generally, shew that

$$(1)\quad \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

$$(2)\quad \sqrt{x}\,\sqrt{y} = \sqrt{xy}$$

where the roots of x and y cannot be extracted. Give an extended definition of multiplication which shall apply to (1) and if possible to (2).

5. Define a surd and examine in what cases the square root of a binomial surd can be expressed in a similar form.

Find the value, to three places of decimals, of the fraction

$$\frac{\sqrt{7} - 2}{\sqrt{11 + \sqrt{112}}}.$$

6. Solve the equation $ax^2 + bx + c = 0$, and discuss the nature of the roots in the different cases.

Shew that if the left hand side of the equation cannot be made negative by substituting any number for x, the equation must have impossible roots.

7. Solve the equations:

$$(1)\quad \frac{x - 2a}{3} = \frac{2x + 6a}{7} - \frac{x + 2a}{13}$$

$$(2)\quad \frac{x - 1}{2} - \frac{x + 1}{3} = \frac{3}{x + 1} - \frac{2}{x - 1}$$

$$(3)\quad \sqrt{x + 4} + \sqrt{2x + 6} = \sqrt{3x + 34}$$

$$(4)\quad x^2(y - 1) + 3y(x^3 - 1) = \sqrt{x^2 + 3y}$$
$$x^2 y = 5.$$

8. From a pyramid of metal on a square base a portion is cut off by a plane parallel to the base. The lower portion could be cast into (1) a complete pyramid on the same base, and height less by $6\frac{3}{7}$ feet than that of the original pyramid; (2) into a rectangular solid on the same base and height $\frac{7}{12}$ of that of the truncated pyramid; (3) into a cube whose edge is 15 feet. Find the dimensions of the first pyramid and the portion of it cut off.

(The volume of a pyramid is one-third of the product of the height and the area of the base.)