STRESSES IN IMPACT*

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THE physical laws of most value in the study of the phenomenon of impact are the laws of conservation of momentum and conservation of energy. By means of the principles expressed by these laws the velocities of two colliding bodies after impact may be determined for all conditions, such as elastic, semi-elastic, and "dead" impact, and the energy lost may be computed when the physical properties of the material are known. Stresses due to suddenly applied loads may also be obtained, with the aid of certain assumptions, when the elastic limit of the material has not been exceeded. In case the elastic limit of the material has been exceeded theory casts little light upon the actual stresses or even approximate stresses such as the modulus of rupture or the stress at failure, which is of so much importance in engineering construction. How do the stresses at failure of a wooden beam, for example, compare with the corresponding stresses for a beam loaded statically to rupture? Will a material absorb more or less work to the point of failure when suddenly loaded than it does for slow loading? Is the modulus of elasticity the same for the two methods of loading? Will a beam deflect farther at rupture in impact than it does in static bending? While the latter question may be readily answered by making a few simple tests, the matter of stresses is not so readily put aside and requires for its solution both the application of the laws of impact and experimental data of a somewhat unique nature. It was primarily the determination of the stresses actually set up in impact that prompted the investigations presented herewith.

Before proceeding to the tests themselves it will be necessary to analyze the phenomena of impact and to formulate the theory involved. In the study of the theory the conditions under which the tests were made will be kept in mind constantly and no assumptions will be made that cannot be amply justified by the results of the test.

For the purpose of this study we will imagine the usual wooden beam supported at the ends and struck at the centre by a falling weight or tup whose mass is at least ten times that of the beam. The beam is rectangular in section, and the nose or surface of contact of the tup is rounded so that undue crushing of the fibres on top of the beam will be avoided. The tup is allowed to fall from a height sufficiently great to break the beam with a single blow. At the instant of contact the pressure between the tup and the beam is zero. Then, as the tup proceeds in its descent, dropping through a distance ΔS, there results, first, a slight depression or indentation in the beam due to the inertia of the particles of the beam in the path of the motion; second, a displacement of the centre of gravity of the section of the beam under the tup equal to ΔY, so that the difference ΔS-ΔY represents the depth of the indentation; and, third, a wave is sent out to each side with a speed equal to that of the velocity of stress propagations in timber. Inasmuch as ΔY is small, the upward pressure of the beam due to flexure is as yet quite negligible, and the actual pressure between the beam and the tup may be considered as due entirely to the inertia of the particles in the vicinity of the centre. As the descent proceeds with ΔS still very small, the difference ΔS-ΔY becomes constant, and soon the centre of gravity of the section under the tup has the same velocity as the tup itself. This does not imply, however, that the sections to either side of the centre have attained velocities proportional to their proximity to the centre or to the deflections associated with a deflection ΔY at the centre, due to static loading. The latter state is merely the limiting or equilibrium condition that the beam assumes as the deflection proceeds. Since the velocity of stress propagation for timber is about 13,000 feet per second, and the total time for the deflection has a minimum value of 0.02 second for the tests made on 50-inch beams, it may be assumed that this condition of equilibrium has been reached a relatively long time before the maximum deflection has been attained. When this condition has been arrived at the beam has an elastic curve very nearly the same as the elastic curve in static bending, and the pressure between the beam and the tup is due solely to flexure. In the meantime, since the bending has increased, the actual pressure between the two has also materially increased, with a corresponding increase in the depression.

Having followed the changes that take place in the beam up to the instant that its inertia has been entirely overcome, we are now in a position to determine the external moments that set up the stresses producing failure.

Considering the forces acting on the tup, there is, first, the force of gravity giving it a downward acceleration g, and, second, an upward force p, the pressure of the beam imparting acceleration in the direction opposite to motion and equal to a. If s stands for the vertical displacement of the tup then $\frac{d^2s}{dt^2}$ represents the rate at which the tup is changing its velocity; that is, the acceleration of the tup, which, it has just been seen, is the resultant of a upward and g downward. Since the motion of the tup relative to the centre of the beam is extremely small, being due only to a change in the indentation, $\frac{d^2s}{dt^2}$ is also the acceleration of the centre beam. Besides these major forces, there remain, of course, friction of the tup in its guides and air resistance. Tests made to find the change in velocity due to friction showed that the velocity was not decreased more than 2 per cent., indicating that friction is quite small when compared to the force of gravity and absolutely negligible when compared with the upward force of the beam.

Put as an equation, these conditions are expressed by the relation

$$\frac{ds^2}{dt^2} = a - g$$

or

$$a = g + \frac{d^2s}{dt^2}$$

If p represents the pressure exerted by the beam upward in pounds, W_t the weight of the tup in pounds, and a and g accelerations in feet per second we have

$$p = \frac{W_t}{g}a = W_t + \frac{W_t}{g}\frac{d^2s}{dt^2} \qquad (1)$$

Proceeding now to the energy-work relations, we obtain the general energy equation:

$$\int (ds - dy)p + \int Fdy + \int \frac{W_b}{2g}u^2 + \tfrac{1}{2}\frac{W_t}{g}v^2 + E_e = \int W_t\, ds + \int \delta W_{bz} + \tfrac{1}{2}\frac{W_t}{g}v_t^2 \qquad (2)$$

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