

The voltage at the generator is, therefore, approximately the same as at the receiver when operating at 55,000 kw. load at 85% power factor. Our assumption that $E_g = E_r$ is therefore very close and it will be unnecessary to recalculate the charging current i_c .

$$\text{From (7) and (8) } .577 \cos \theta = \frac{104125}{127000} = .82 \quad \theta = 35^\circ \quad \sin \theta = \frac{73300}{127000}$$

$$\text{From (9) } I_g = \sqrt{(31.1 - 45.3 \times .577)^2 + (45.3 \times .82)^2} = \sqrt{4.9^2 + 37.1^2} = 37.5 \text{ amperes.}$$

$$\text{From (10) } \sin \alpha = \frac{45.3 \times .82}{37.5} = .99, \alpha = 82^\circ.$$

$$\text{From (11) } \beta = \theta - \alpha = 35^\circ - 82^\circ = -47^\circ, \cos \beta = .682.$$

The minus sign denotes a negative angle of lag or an angle of load, indicating that with a load of 5,500 kw. at 85% power factor lagging at the receiver end, the charging current is sufficient to give a leading current of 68.2% power factor at the generator.

$$\text{From (12) Loss in line} = [31.1^2 + 31.1(37.5 - 31.1) + \frac{1}{3}(37.5 - 31.1)^2] \sqrt{3} 68.2 = 138.5 \text{ KW.}$$

Check on 44,000 kw. Load at Receiver at 85% Power Factor. $I_r = 249$ Amperes.

$$\text{From (5) } E_g^1 = \sqrt{(120,000 \times .85 + 249 \times 68.2)^2 + (120,000 \times .53 + 249 \times 316)^2} = \sqrt{(119,000)^2 + (142,300)^2} = 185,500 \text{ volts.}$$

$$\text{As a first approximation assume that } E_g = E_r \text{ and as before, see (4) then, } I_c = \frac{2\pi \times 60 \times 1.735}{\sqrt{3} \times 10^6} \times 120,000 = 45.3 \text{ amperes.}$$

$$\text{From (6) } E_g = 185,500 - \frac{45.3}{2} 316 = 185,500 - 7,150 = 178,350 \text{ volts.}$$

Our assumption that $E_g = E_r$ is therefore incorrect, so we must recalculate i_c from (4) and substitute again in (6) to get E_g .

$$i_c = \frac{2\pi \times 60 \times 1.735}{\sqrt{3} \times 10^6} \times \frac{178,350 + 120,000}{2} = 56.2 \text{ amperes.}$$

$$\text{Substituting again in (6) } E_g = 185,500 - \frac{56.2}{2} 316 = 185,500 - 8,900 = 176,600.$$

This compares very closely with $E_g = 177,300$ as obtained by Peek from his more accurate formulas.

$$\text{From (7) and (8) } \sin \theta = \frac{142,300}{185,500} = .766 \quad \cos \theta = \frac{119,000}{185,500} = .641 \quad \theta = 50^\circ.$$

$$\text{From (9) } I_g = \sqrt{(249 - 56.2 \times .766)^2 + (56.2 \times .641)^2} = \sqrt{208.7^2 + 36.1^2} = 212 \text{ amperes.}$$

$$\text{From (10) } \sin \alpha = \frac{56.2 \times .641}{212} = .17 \quad \alpha = 9.78^\circ.$$

$$\text{From (11) } \beta = \theta - \alpha = 50 - 9.78 = 40.22^\circ \quad \cos \beta = .765 = \text{power factor at generator.}$$

The plus sign for β indicates a lagging current at the generator.

$$\text{From (12) Loss in line} = [212^2 + 212(249 - 212) + \frac{1}{3}(249 - 212)^2] \sqrt{3} 68.2 = 6,300 \text{ KW.}$$

Check on no Load.

Assume as a first approximation that the voltage at the generator and receiver ends of the line are equal. As a first approximation we have as charging current from (4)

$$i_c = \frac{2\pi \times 60 \times 1.735}{\sqrt{3} \times 10^6} \times 120,000 = 45.4.$$

Since there is no load on the receiver, $I_r = 0$ and (6) becomes,

$$E_g = E_r - \frac{i_c}{2} x = 120,000 - \frac{45.4}{2} \times 316 = 120,000 - 7,180 = 112,820 \text{ volts.}$$

Now recalculates i_c from (4),

$$i_c = \frac{2\pi \times 60 \times 1.735}{\sqrt{3} \times 10^6} \times \frac{112,820 + 120,000}{2} = 44.1 \text{ amperes.}$$

Recalculating the generator voltage from (6),

$$E_g = 120,000 - \frac{44.1}{2} 316 = 120,000 - 7,000 = 113,000.$$

$$\text{Line loss} = \frac{1}{3} (44.1^2 \sqrt{3} 68.2) = 76.5 \text{ KW.}$$

The following table gives the comparison of the results obtained by Mr. Peek's formulas and the approximate formulas as worked out on the transmission line treated above. It will be noted that the difference between the results obtained by the two methods is in most cases very small, and in other cases close enough for practical purposes.

	No Load.	5,500 K.W., P. F. .85.	44,000 K.W., P. F. .85.
	Approx. Peek.	Approx. Peek.	Approx. Peek.
Gen. Kilovolts	113	112.9	119.85
Gen. Amperes	44.1	44.42	37.5
Line Loss KW.	76.5	81.36	138.5
P. F. at Generator	0	0094	68.2
			69.9
			76.5
			76.6

Derivation of the Transmission Formulas.

Formulas (1), (2) and (3) are the usual ones employed for transmission lines, and need no explanation. The capacity is treated as if a condenser were concentrated in the centre of the line, so that the charging current flows through only half of the impedance of the line. It is considered that the voltage across the condenser is equal to the average of the voltage at the generator and receiver ends of the line or $\frac{E_g + E_r}{2}$. The charging current is,

therefore, expressed as in formula (4). It is assumed that the phase position of the charging current is 90° ahead of the generator voltage E_g . This is not strictly correct, since for that portion of the line nearest the generator the charging current is 90° ahead of E_g , while for that portion nearest the receiver the charging current is 90° ahead of E_r . The voltages E_g and E_r are not in phase but differ by the angle $\phi - \theta$.

If there were no capacity in the transmission line the voltage at the generator end would be expressed in the usual way by formula (5). However, if capacity cannot be neglected there must be added to E_g the voltage consumed by the charging current flowing through the impedance of the line. This voltage may be split into two

components — $\frac{i_c}{2}$ in phase with E_g^1 and the component $\frac{i_c}{2} R$ in quadrature with E_g^1 . We should therefore write

$$E_g = \left\{ \left[\sqrt{(E_r \cos \phi + I_r R)^2 + (E_r \sin \phi + I_r x)^2} - \frac{i_c}{2} x \right]^2 + \left[\frac{i_c}{2} R \right]^2 \right\}^{1/2}$$

The second term $\frac{i_c}{2} R$ can be neglected so that the

equation will take the form shown in formula (6). Neglecting this last term bring E_g and E_g^1 in phase, as shown in the figure. This is not strictly correct, but the error resulting from this assumption is small. The sine and