geneous or not, the number of terms in a complete function, and some of the simpler properties of each kind, form the elementary part of what Chrystal terms the study of mathematical form, and Sylvester speaks of as morphology. It is scarcely too much to say that an average schoolboy first comes to have a dim idea of integral rational functions by noticing how many of the examples he works have expressions with indices going regularly up and down, and he looks upon it as a rather pleasant and kindly device of the writer of his book to make the sums come out neatly. As to classification of expressions he knows nothing. It may be fanciful, but I am often tempted to encourage pupils to ascribe, not merely form, but something like moral and social qualities, to certain functions. Thus $a^2 - b^2$ may be compared to an affable courteous man, ready at all times to do you as much service as he can; $a^2 + b^2$ to one who is difficult of access, somewhat austere and forbidding in appearance, but from whom, when you really get to know him, you will learn much, and have your mind enlarged and improved; $a^3 + b^3 + c^3 - 3abc$ as a rather eccentric person, who is constantly turning up in unexpected places, asking whether you do not remember to have met him before.

In all the rudimentary work one important matter is constantly—I might say universally — neglected. That matter is verification. A boy has made a great step to the front in his mathematical studies when he honestly recognizes the fact that he sometimes makes a mistake. Revision, searching for possible error, do not come naturally to a young learner. He must be trained to practise these checks on his work, as when left to himself, he acts as if he were a small Pope in respect of infallibility. In arithmetic there is the test of correctness by casting out the nines, the one valuable process we owe to the Arabs; and this is more often neglected than used in school teaching. In algebra there is the test of substituting 1 for x in the work, and seeing whether the result is what it ought to be; and, speaking roughly, it may be said that never is this check taught in schools.

The examination of the form of the products of the factors, $x+a$, $x+b$, $x+c$, etc., and of $x-a$, $x-b$, $x-c$, etc., should be undertaken when multiplication is being taught. Here is almost the sole occasion on which I feel inclined to depart from the order in Chrystal's "Algebra," adopting in preference that in Clifford's "Common Sense of the exact sciences." I think we should here bring to our assistance the first principles of permutations and combinations. The value of the results thus obtained is very great, as giving the means of profuse illustrations of the principles of symmetry, as affording an additional check on accuracy of work, as enabling the pupil to forecast the number of terms he will have to deal with in any process, and as familiarizing him with the valuable symbol Σ, as the sum of terms formed under like conditions. These results are obtained by the purest arithmetic from elementary commonsense principles, without even the idea of a fraction being required. They lead at once to the binomial theorem so far as a positive integral index is concerned.

Some idea of a limit ought to be given at an early stage of the pupil's progress. It is customary at present to look upon circulating decimals as a part of arithmetic which is of little practical importance, and on which much time and attention should not be spent. Nevertheless, the first rudiments of circulating decimals are valuable, if only for the sake of illustrating the nature of a limit. I venture to think that the tendency to