

tions will be upon a straight line with the ultimate intersection n' ; which arc shall meet the arc CD, between the points K and D.—Also there shall be another arc of the radius A through B, upon which the intersections in the same manner described, shall be in a straight line with the ultimate intersection n'' —which arc must meet the arc CD, between the points K' and C.

For the points 1, 2, 3 ... $4\frac{3}{2}$, have varied by construction to be the points d, s, r, ... q, o, K on the arc BK, and the intersection, $a'b'e' \dots n$ to be the intersections V, p, q, ... n' , but in this variation the curve $a'b'e' \dots n$, has passed over to the opposite side of its chord and become the curve Vpq ... n' , and therefore (Lem. 4.) there must be an arc of the radius AC through B, between the arcs BK and BD, upon which the chord and curve must be on a straight line—also by the same, there must be another arc between the arc BK' and BC, on which the curve and its chord must be on one straight line.

NOTE.—The side of the chord on which may be the convexity or concavity, of a curve line drawn through intersections as above described, must evidently depend upon the law of the differences or distances of the points, from the point of ultimate intersection, and also the distance of the point n from the centers B and D, and as these may be varied, the chord line may be on the one side or the other of its curve, or the concavity of the curve changed to the opposite sides. For the points of variation upon which the chord and curve would be on one straight line, are unknown, and therefore the side of the curve on which the chord is, can only be known by construction.

LEMMA 6.

FIG. 6. Draw through the points of intersections B, d, e, f ... h, i, k, D', the curved line Bdef ... hikD', and from the point B as a center with the distances Bk, Bi and Bh, &c.,—and from the point D' as a center, with the distances D'd, D'e, and D'f, describe the intersections a, b, and c &c., and through a, b, c &c., draw the line abc ... n, and let it be granted that the curve line abc ... n is through an infinite number of points, meeting the curve line Bdef ... hikD' in the point n, and by construction is concave to the arc nB, and consequently the chord an must be on the opposite side of the curve line abc ... n, to what that of the chords Vn' and Vn'' (Fig. 5.) is to their curve lines Vpq ... n' and $V'p'q' \dots n''$; There shall be an arc or curve line similar and equal to the curve line Bdef ... hikD' through B, between the curve Bdef ... hikD' and the arc or curve line Bd'sr ... qoK (Fig. 5.) so that on its intercepted arc between the arcs 2'K and IT, the curve line Vpq ... n' , and its chord Vn' , will be on the same straight line,—also there shall be an arc or curve line similar and equal to the curve line Bdef ... hikD', through B, between the arc or curve line Bduv ... q'oK', (Fig. 5.) on its intercepted arc, between the arcs 2'K' and IT', so that the curve line $V'p'q' \dots n''$ and its chord $V'n''$ will come to be on the same straight line.

For it is evident, by the construction that the curve line Bdef ... hikD' is the only common arc which can be described intersecting the series of arcs 1K and 1'K', therefore there cannot be any common intercepted arc but dk—but there must be two intercepted arcs (Lem. 5.) on which the curves Vpq ... n' and $V'p'q' \dots n''$ would coincide with their chords—therefore the one must be on an arc through B, between Bdef ... hikD' and Bd'sr ... qoK, and the other through B, must be on an arc between the arc Bduv ... q'oK' and the curve Bdef ... hikD'.

LEMMA 7.

FIG. 6 From the point B as a center, with the distances Bk, Bi, Bh, and from the point D' as a center, with the distances D'e, D'f, D'g, describe the intersections n' , b', and c',* and draw the line $a'b'e' \dots n'$ and join an' (Lem. 4)—but the line $a'b'e' \dots n$ is a curve line by construction, and concave to the arc $n'D'$, and therefore upon the opposite side of its chord an , to that of the line abc ... n to its chord an .—There shall be points on

* The intersection c' cannot be introduced on the figure, without describing it on a much larger scale.