

## \* Mathematics. \*

All communications intended for this department should be sent before the 20th of each month to Chas. Clarkson, B.A., Seaforth, Ont.

## CORRESPONDENCE.

S.F., Petrolia.—The whole of McLellan's Elementary Algebra is required for Second Class—Elementary rules; factoring; H.C.M.; L.C.M.; square root; fractions; ratio; simple equations of one, two and three unknown quantities; indices and surds; quadratic equations.

F.E.F.—You have not given the problems and references to the text-books, as our rule requires. Attend to this next time.

47. "The dividend is 2547346; the remainder is 2654 less than the divisor; find the divisor." The data appear to be insufficient.

48. Pub. Sch. Arith., p. 112, question 40. The number is the least multiple of 120, which is less by 15 than a multiple of 25. Taking 120, 240, 360, etc., we see that 375 is the least.

49. "Counting the eggs by 2's, 3's, 4's, 5's or 6's there is always a remainder of 1; but counting by 7's there is no remainder. Find the least number of eggs possible." L.C.M. of 2, 3, 4, 5, 6=60. No. required is a multiple of 60 that is less by 1 than some multiple of 7. Ans.—301.

50. "A boy spent 20 cts. for 20 pencils, some at 4c., some at 3c. and some at 1/2c. each. How many of each kind did he buy?" The average is 1c. each, so we have 3, + 1/2, + 1/2, i.e., 12, + 2, + 3 as the differences from the average. Make the losses cancel the gains. Take 3 at 4c., 15 @ 1/2c., and 2 @ 1/2c.; 20 for 20 cts.

51. H. Smith's Arith., p. 199, IV. 5. Take B's flour as the standard of quality; then the quality of A's, B's, C's flour are as 55 : 50 : 58; and taking into account the quantities the money must be divided as 55 × 125 : 50 × 150 : 58 × 225. And the money to be divided is 500 × 672. The proportions reduce to 275 : 300 : 522; hence A's share = 5 × 672 ÷ 1097 = \$842.301, etc., for B and C.

S.F.P.—52. "A person buys 6% bonds, the interest on which is payable yearly and which are to be paid off at par 3 years after the time of purchase. If he invests his interest when received at 4% compound interest, what should he pay for the bonds to realize 7% compound interest on his money?"

Supposing the bond is for \$100, its amount at the end of three years = 100 + 6(1.04<sup>2</sup> + 1.04 + 1). Let x = price to be paid, then at 7% compound interest, this will amount to x(1.07)<sup>3</sup> at the end of the third year. Hence the equation,

$$x(1.07)^3 = 100 + 6(1.04^2 + 1.04 + 1);$$

$$\text{and } x = \frac{106 + 6(1.04^2 + 1.04)}{1.07^3} = \text{etc.}$$

$$52. \frac{5x^2 + x - 3}{5x - 4} - \frac{7x^2 - 3x - 9}{7x - 10} = \frac{x - 3}{35x^2 - 78x + 40}$$

Take the fractions on the left together; sum = 0, and the denominator is the same as the denominator on the right.

$$\therefore x - 3 = 0, x = 3.$$

$$53. \frac{5}{x-1} + \frac{4}{x+2} + \frac{21}{x-3} = \frac{5}{x+1} + \frac{4}{x-2} + \frac{21}{x+3}$$

Combine in this way:

$$5\left(\frac{1}{x-1} - \frac{1}{x+1}\right) + 4\left(\frac{1}{x+2} - \frac{1}{x-2}\right)$$

$$+ 21\left(\frac{1}{x-3} - \frac{1}{x+3}\right) = 0$$

$$\text{or, } 5\left(\frac{2}{x^2-1}\right) - 4\left(\frac{4}{x^2-4}\right) + 21\left(\frac{6}{x^2-9}\right) = 0$$

$$\text{i.e., } \frac{5}{x^2-1} - \frac{8}{x^2-4} + \frac{63}{x^2-9} = 0$$

$$\therefore x^4 - 5x^2 + 6 = 0, x = \pm \sqrt{2} \text{ or } \pm \sqrt{3}.$$

$$54. x^4 - 4\frac{1}{3}x^3 + 5\frac{1}{3}x^2 - 4\frac{1}{3}x + 1 = 0$$

$$\therefore x^2 - 4\frac{1}{3}x + 5\frac{1}{3} - 4\frac{1}{3}x^{-1} + x^{-2} = 0$$

$$\therefore (x^2 + x^{-2}) - 4\frac{1}{3}(x + x^{-1}) + 5\frac{1}{3} = 0$$

$$\therefore (x + x^{-1})^2 - 4\frac{1}{3}(x + x^{-1}) + 2\frac{1}{3} = 0, \text{ a quadratic.}$$

J.H.F., Wiarton, solves No. 33 as follows:

Given  $(x+y)z = a$ ;  $(z+x)y = b$ ;  $(y+z)x = c$ . Add the three and we get  $xy + yz + zx = \frac{1}{2}(a+b+c)$ ; take the first from this and  $xy = \frac{1}{2}(b+c-a)$ , and  $yz$  and  $zx$  are symmetrical with this result. Hence  $yz \times zx \div xy = z^2 = \frac{1}{2}(c+a-b)(a+b-c) \div (b+c-a)$ .  $\therefore x^2$  and  $y^2$  by symmetry. He wishes a solution of this question:

55. "If  $\alpha_1\beta$  are the roots of  $x^2 + px + q = 0$ , and  $\alpha_1, \beta_1$  are the roots of  $x^2 - p_1x + q_1 = 0$ , then  $\alpha_1\beta + \beta_1\alpha$  and  $\alpha\beta + \alpha_1\beta_1$  are the roots of the equation  $x^2 - pp_1x + p^2q_1 + p_1^2q - 4qq_1 = 0$ ."

We must show that  $pp_1 = (\alpha_1\beta + \beta_1\alpha) + (\alpha\beta + \alpha_1\beta_1)$  and that

$$p^2q + p_1^2q - 4qq_1 = (\alpha_1\beta + \beta_1\alpha)(\alpha\beta + \alpha_1\beta_1); \quad (A)$$

or,  $pp_1 = \alpha + \alpha_1)(\beta + \beta_1)$ . But  $\alpha + \beta = -p$ ;  $\alpha_1 + \beta_1 = -p_1$

$\therefore -pp_1 = (\alpha - \beta)(\alpha_1 + \beta_1)$  and this is manifestly not identical with the required result unless  $\alpha_1 = \beta$ , and the second equation is made  $x^2 - p_1x + q_1$ , or else the last equation  $x^2 + pp_1 + \text{etc.}$  In the second part we see that  $4qq_1 = \alpha\beta\alpha_1\beta_1$ , and

$$p^2q_1 + p_1^2q \text{ gives } \alpha_1\beta_1(\alpha^2 + \beta^2) + \alpha\beta(\alpha_1^2 + \beta_1^2) + 4\alpha\beta\alpha_1\beta_1$$

or  $p^2q_1 + p_1^2q - 4qq_1 = \alpha_1\beta_1(\alpha^2 + \beta^2) + \alpha\beta(\alpha_1^2 + \beta_1^2)$ , which does not agree with the product (A) unless  $\alpha_1 = \beta$ . It seems that in some way the question is imperfectly stated.

S.I.—Your problem seems to involve one of the higher curves and to lie beyond the ordinary limits of this column. Perhaps some reader may find a suitable solution; we give the problem:

56. A pole 100 feet high and 1 foot in diameter at the base, and 1 inch in diameter at the top, has a vine twined around it. The circles made by the vine are 1 foot apart. What is the length of the vine?

57. By Zeno, Shelburne.—We strongly suspect that your problem is of the same class as No. 56. If any ingenious reader can calculate the length of the carpet, here is the problem:—"A carpet 3 feet wide is laid diagonally in a room 40 × 13 feet so that each corner of the carpet touches a side of the room. The carpet is cut off square, find its length." Practically the problem can be solved most easily by drawing the figure to scale and reading off the length; we do not perceive any elementary solution by calculation, but we invite our friends to search for one.

G.W.D., Marsh Hill.—1. "Is a teacher justified in saying that there are at least twenty incorrect answers in the Public School Arithmetic?" Very likely; it requires extraordinary care and labor to get mathematical copy set up accurately. Probably the second edition will be revised and corrected. You ought to point out the errors to the publishers. 2. "Is it necessary for pupils to show full work on Entrance Examination, or will the shortest way possible be accepted?" We think that every pupil ought to put down his work articulately and in good, clear order, not crowded together. The method of doing the question should certainly be indicated, so that in case of any slight mistake the examiner may be able to give the candidate full credit for knowing how to do the question, although he may have made a small slip in the execution of it. 3. "In papering walls, why is not the height of the room taken into account?" Probably you refer to p. 78 of the P. S. Arith. The height is there taken into account—"a room of ordinary height." The page is rather obscure, however, and might easily have been made more precise.

58. See P. S. Arith., p. 146, No. 28. The average time of arrival is the average of 10 " 15" " 0"; 10 " 10" " 30" etc. This is 10 " 12" " 40", from which take 3" " 15" to get the schedule time.

59. See P. Sch. Arith. p. 151, No. 102. HINT.—Keep separate accounts for the water and the vinegar in each vessel, thus, at the end of the first stage A<sub>1</sub> 0; B 1, 1; C 4, 1. At the end of second stage A<sub>1</sub> 0; B 1 $\frac{1}{2}$ , 1 $\frac{1}{2}$ ; C 2 $\frac{1}{2}$ , 1 $\frac{1}{2}$ ; and at the end of the first stage

A<sub>1</sub> 0; B 2 $\frac{1}{2}$ , 2 $\frac{1}{2}$ ; C 3 $\frac{1}{2}$ , 3 $\frac{1}{2}$  where 3 $\frac{1}{2}$  is the vinegar and 3 $\frac{1}{2}$  the number of gallons.

60. By A SUBSCRIBER, Simcoe.

A market woman has an exact number of dozens of eggs. She finds that she can count them by 8 or by 10 or by 20, always having 4 eggs over. Find the least number of dozens she can have?

SOLUTION.—L.C.M. of 8, 10, 20=40, hence we must find the least multiple of 40 that with 4 added becomes a multiple of 12; but of 44, 84, 124, 164, etc., 84 is the least will contain 12. Ans.—Seven dozen.

61. By the same. See No. 48 above.

REMARK.—The mass of correspondence to be handled this month has prevented the appearance of the solutions originally intended for this issue. We are glad to find out the needs of our patrons and to supply them as far as possible. What is easy and plain to one may seem difficult to another; the Editor of this Department wishes to be useful to working teachers in the first place, and secondly to those who are going up for examinations, and thirdly, to lovers of mathematics generally. Will all our friends make an effort to bring the claims of THE JOURNAL impressively before the minds of non-subscribers?

## Book Notices, etc.

Any book here reviewed sent post-paid on receipt of price. Address The Grip Printing & Publishing Co., Toronto.

*Euripides, Medea.* M. A. Bayfield, M.A. Macmillan's series of Elementary Classics.

Admirable as most of the books of this series. The scenic explanations add a great deal of interest. Mr. Bayfield's name renders further comment needless.

*Papers read before the Mathematical and Physical Society of Toronto University during the year 1890-91.* Toronto: Rowse & Hutchison.

This booklet contains five papers of very great interest to every one at all conversant with the leading problems of physical science. Prof. Baker's paper on "Poetic Interpretation in Mathematics," and Mr. Chant's on "The Structure of Matter," will appeal to the taste of all educated people, even those who do not burn incense at the shrine of mathematics.

*Promissory Notes, Drafts and Cheques. What a Business Man Should Know Regarding Them.* By J. W. Johnson, F. C. A., Principal Ontario Business College, Belleville. Published by Ontario Business College, Belleville.

This is the third edition of this useful and popular manual, the substance of which first appeared in the EDUCATIONAL JOURNAL in 1888. The subject, as indicated by the title, commends itself as one of much interest both to teachers and to business men. The fact that the author has been for fourteen years lecturing upon this and kindred topics in Ontario Business College, and that the little book has stood the test of publication and has now reached a third edition, are sufficient guarantee of the merits and value of the work.

## SHEPP'S PHOTOGRAPHS OF THE WORLD.

This is one of the most beautiful and complete picture albums ever published. It contains more than five hundred full page photographs of "street scenes, cathedrals, mosques, temples, castles, obelisks, pyramids, active volcanoes, mountain, lake and river scenery, with instantaneous pictures, showing the every-day life of the people in various countries, and direct copies of the most famous paintings and statuary in the leading galleries of Europe." There are scenes not only from the great cities and famous resorts of Europe and America, but from Egypt, Palestine, Syria, India, China, Australia and other remote countries. The photographs are at once accurate and artistic, and in many cases seem to transport the beholder at once to the scenes and historic remains of distant lands "old in story." To the few who have been privileged to travel amid the scenes here depicted, these graphic pictures will recall delightful memories, while to the many who can never hope to indulge