(If AB, AC be the given straight lines, draw DE parallel to AC, cutting AB in D, and at a distance from AC equal to the given constant length. The line bisecting the angle BDE is the required locus.)

3. Find the locus of the middle points of parallel chords in a circle. (A diameter.)

4. Find the locus of the middle points of equal chords in a circle. (A concentric circle.)

5. The locus of the vertices of all triangles on the same base and having the same vertical angle is a circle. (Prop. 21, Bk. III.)

6. If from two fixed points in the circumference of a circle, straight lines be drawn intercepting a given arc and meeting without the circle, the locus of their intersections is a circle. (It may readily be shown that the angle at the intersection of the lines is constant, and the line joining the fixed points is constant; hence by previous problem the locus is a circle.)

7. P is any point in a semicircle whose diameter is AB; AP is produced to Q, so that PQ is equal to PB. Find the locus of Q. (Evidently a segment of a circle containing an angle of 45°, described on AB.)

8. If from any external point any number of straight lines be drawn cutting a circle, find the locus of the middle points of the chords thus formed. (If A be the external point, and B the centre of the circle, the locus is a portion of the circle described on AB as diameter.)

9. Given the base and the sum of the squares on the sides of any triangle, find the locus of its vertex. (Use Prob. 145, page 351, Todhunter's Euclid, or Prob. 1, page 91, Hamblin Smith's Euclid. Construct a square equal to the difference between half the sum of the squares and the square on half the given base. With the side of this square as radius and the middle point of the given base as centre, describe a circle. This circle will be the locus required;—seen from the props. we have referred to.)

10. The base of a triangle and the radius of its circumscribing circle being given, find the locus of its vertex. (On the given base construct a triangle with sides each equal to the given radius. The vertex of this triangle is the centre of the circumscribing circle, any point on the circumference of which may be the vertex of the required triangle.)

11. If a circle roll within another of twice its size, any point in its circumference will trace out a diameter of the first. (Let A be the point in the large circle from which P the point in the small circle starts, and C the centre of the large circle. And at any time during the rolling assume that P is in AC, and let B be the present point of contact. Then the angle subtended at the centre of the small circle by BP is double the angle subtended at the centre of the large circle by AB; and hence, one circle being double the other, the arc BP is equal to the arc BA, and this is the condition for rolling.)

12. Find the locus of a point, such that if straight lines be drawn from it to the four corners of a given square, the sum of the squares shall be invariable. (The required locus is a circle whose centre is the intersection of the diagonals of the square, the particular circle being determined in a given case by the given sum of the squares on the lines drawn to the corners of the square.)

INTERSECTION OF LOCI.

When a point has to be found which satisfies *two* conditions, the problem is generally determinate if it is possible: and the method of loci is very frequently employed in discovering the point. For if the locus which satisfies each condition separately be constructed, the intersection or intersections of these loci will mark the point or points at which both conditions are satisfied.

Applications of the principle of intersection of loci in determining problems are frequently met with; *e.g.*, Prop. 1, Bk. I. Here it is required to determine a point subject to the two conditions that its distances from two given points A and B shall be equal to a given distance. Accordingly Euclid constructs the locus of points at the given distance from A; then the locus of points at the given distance from B; the intersections of these loci give points satisfying the required conditions. Prop. 22, Bk. I., is another example.

The following are additional illustrations:—

1. Find a point in a given straight line at equal distances from two given points.

2. Find a point in a given straight line at a given distance from a given straight line.

3. Find a point in a given straight line at equal distances from two given straight lines.

4. Describe an isosceles triangle on a given base, its sides being of a given length.

5. Find a point at a given distance from a given point, and at the same distance from a given straight line.

6. Given the base, the sum of the sides, and one of the angles at the base of a triangle, construct the triangle.

7. Given the base, the difference of the sides, and one of the angles at the base of a triangle, construct the triangle.

8. Find a point at given distances from the circumference of two given circles.

J. P. has sent in a correct solution of Prob. 4, March number.

S., of Woodstock, has forwarded one solution of Prob. 15, May number.

Mr. G. Shaw, of Kemble, has correctly solved Prob. 17, May number.

We are asked for the solution of Question 4, page 218, Hamblin Smith's Arithmetic.

The total cost is \$1700000; the amount of stock \$1500000; leaving an assessment of \$200000 to be made on the shareholders. This is $13\frac{1}{3}$ per cent. on \$1500000, which, with the final call of 30 per cent., makes $43\frac{1}{3}$ per cent.

Mr. J. J. Magee, M. A., Uxbridge, sends the following questions with solutions, the solutions having been asked for at the late Ontario Co. Teachers' Association:

1. If a person spends $\frac{3}{7}$ of his money, and \$20 more than $\frac{7}{9}$ of the remainder less \$20, and has \$28 left, how much had he at the beginning?

If he spends $\frac{3}{7} + 20$, he has $\frac{4}{7} - 20$ left.

.... $\frac{7}{9}$ of this $- 20$, he has $\frac{2}{9} \div 20$ left;

$\therefore \frac{2}{9}\left(\frac{4}{7} - 20\right) + 20 = 28,$

or $\frac{8}{63} = 12\frac{4}{9}$, or unity $=$ \$98.

Otherwise, $\frac{2}{9} + 20 = 28$, $\frac{2}{9} = 8$, unity $= 36$, what he had left after first expenditure. $\therefore \frac{4}{7} - 20 = 36$, $\frac{7}{7} = 98$, as before.

2. Two persons, A and B, gain \$700. A's money was 3 months in trade, and his gain was \$300 less than his stock. B's, which was \$250 more than A's, was in trade 5 months. Find A's stock.

Let $x = A$'s stock; $x + 250 = B$'s. $\frac{x - 300}{3x} =$ gain on \$1.00 for 1 month, found from A's gain. Similarly $\frac{700 - (x - 300)}{5\,(250 + x)} =$ same from B's gain. Equating these we have a quadratic, giving A's stock \$500.

3. Question 5, page 239, McLellan & Kirkland's Examination Papers.

$\frac{48}{500}$ males $-$ $\frac{23}{500}$ females $= \frac{8}{500}$ (males $+$ females). Thence males $\times\ 40 =$ females $\times\ 32$; or males are to females as 4 is to 5.