Appendix

Derivation of the Binomial Probability Model[1]

The binomial probability density function is:

$$P(x) = \frac{n!}{x!\,(n-x)!}\,p^x\,(1-p)^{n-x} \qquad \text{(i)}$$

where x = number of successes;
n = number of independent trials;
p = probability of success in each trial.

The binomial probability model presented on p. 30 calculates the probability of **at least one** detection of a treaty violation. In other words,

$$\begin{aligned} P(\text{at least one detection}) &= 1 - P(\text{no detections}) \\ &= 1 - P(0) \end{aligned}$$

For P(0), Equation (i) becomes

$$P(0) = \frac{n!}{0!\,(n-0)!}\,p^0\,(1-p)^{n-0}$$

$$= \frac{n!}{n!}\,(1-p)^n \qquad [0! = 1;\ p^0 = 1]$$

$$= (1-p)^n \qquad \text{(ii)}$$

Therefore,

$$\begin{aligned} P(\text{at least one detection}) &= 1 - P(\text{no detection}) \\ &= 1 - (1-p)^n \qquad \text{(iii)} \end{aligned}$$

Note

1. I am grateful to Mr. Ed Emond, Directorate of Mathematics and Statistics, Department of National Defence, Ottawa, Canada for his explanation of the binomial probability model and the derivation presented here. Responsibility for the application of this model to the verification problem in conventional arms control rests solely with the author.