(2) $a=xy^{p-1}$ $\therefore$ $a^{q-r}=x^{q-r}\cdot y^{(p-1)(q-r)}$,

$b=xy^{q-1}$ $\therefore$ $b^{r-p}=x^{r-p}\cdot y^{(q-1)(r-p)}$,

$c=xy^{r-1}$ $\therefore$ $c^{p-q}=x^{p-q}\cdot y^{(r-1)(p-q)}$,

$\therefore$ $a^{q-r}\cdot b^{r-p}\cdot c^{p-q}=$

$$x^{(q-r)+(r-p)+(p-q)}\cdot y^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$$
$$=x^0\cdot y^0=1.$$

7. Cube

$$\left(-\frac{r}{2}+\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)^{\frac{1}{3}}+\left(-\frac{r}{2}-\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)^{\frac{1}{3}}.$$

$$\left\{\left(-\frac{r}{2}+\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)^{\frac{1}{3}}+\left(-\frac{r}{2}-\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)^{\frac{1}{3}}\right\}^3=x,$$

$$\left(-\frac{r}{2}-\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)+\left(-\frac{r}{2}+\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)$$

$$3\sqrt[3]{\frac{r^2}{4}-\left(\frac{r^2}{4}+\frac{q^3}{27}\right)}\left\{\left(-\frac{r}{4}+\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)^{\frac{1}{3}}+\left(-\frac{r}{2}-\sqrt{\frac{r^2}{2}+\frac{q^3}{27}}\right)^{\frac{1}{3}}\right\}$$

$$=-r-q\left\{\left(-\frac{r}{4}+\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)^{\frac{1}{3}}+\left(-\frac{r}{2}-\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)^{\frac{1}{3}}\right\}=-r-qx.$$

8. Extract the square root of

(1) $\dfrac{1+4x^{\frac{1}{2}}-2x-12x^{\frac{3}{2}}+9x^2}{1-4x^{\frac{1}{2}}+6x-4x^{\frac{3}{2}}+x^2}$.

(2) $1+a^2+\sqrt{1+a^2+a^4}$.

$$(1)=\left\{\frac{1+4x^{\frac{1}{2}}-2x-12x^{\frac{3}{2}}+9x^2}{1-4x^{\frac{1}{2}}+6x-4x^{\frac{3}{2}}+x^2}\right\}^{\frac{1}{2}}$$

$$=\left\{\frac{(9x^2-6x+1)-4x^{\frac{1}{2}}(3x-1)+4x}{(x^2+2x+1)-4x^{\frac{1}{2}}(x+1)+4x}\right\}^{\frac{1}{2}}$$

$$=\left\{\frac{(3x-1)^2-4x^{\frac{1}{2}}(3x-1)+4x}{(x+1)^2-4x^{\frac{1}{2}}(x+1)+4x}\right\}^{\frac{1}{2}}$$

$$=\frac{3x-1-2x^{\frac{1}{2}}}{x+1-2x^{\frac{1}{2}}}=\frac{(3x^{\frac{1}{2}}+1)(x^{\frac{1}{2}}-1)}{(x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-1)}=\frac{3x^{\frac{1}{2}}+1}{x^{\frac{1}{2}}-1}.$$

(2) Let $\left\{1+a^2+\sqrt{1+a^2+a^4}\right\}^{\frac{1}{2}}=x^{\frac{1}{2}}+y^{\frac{1}{2}}$ (1)

then always will

$$\left\{1+a^2-\sqrt{1+a^2+a^4}\right\}^{\frac{1}{2}}=x^{\frac{1}{2}}-y^{\frac{1}{2}}\ (2)$$

By multiplying (1) by (2) we obtain $x-y=a$, and by squaring (1) we obtain $x+y=2+a^2$,

$$\therefore\ x=\frac{a^2+a+1}{2}\text{ and }y=\frac{a^2-a+1}{2},$$

$$\therefore\ \left\{1+a^2+\sqrt{1+a^2+a^4}\right\}^{\frac{1}{2}}$$
$$=\pm\sqrt{\frac{a^2+a+1}{2}}\pm\sqrt{\frac{a^2-a+1}{2}}.$$

9. Determine the values of m which make the expression $3mx^2+(6m-12)x+8$ a complete square.

In order that this may be a complete square, $96m$ must equal $(6m-12)^2$,

that is, $3m^2-20m+12=0$,

" $(3m-2)(m-6)=0$,

$\therefore$ m may be 6 or $\frac{2}{3}$.

10. Express $\dfrac{5^{\frac{1}{2}}-7^{\frac{1}{4}}}{5^{\frac{1}{2}}+7^{\frac{1}{4}}}$ by an equivalent fraction with rational denominator.

By multiplying the numerator and denominator by $(5^{\frac{1}{2}}-7^{\frac{1}{4}})(5+7^{\frac{1}{2}})$ the fraction becomes

$$\frac{(5^{\frac{1}{2}}-7^{\frac{1}{4}})(5^{\frac{1}{2}}-7^{\frac{1}{4}})(5+7^{\frac{1}{2}})}{(5^{\frac{1}{2}}+7^{\frac{1}{4}})(5^{\frac{1}{2}}-7^{\frac{1}{4}})(5+7^{\frac{1}{2}})}$$
$$=\frac{16-5\cdot5^{\frac{1}{2}}\cdot7^{\frac{1}{4}}+5\cdot7^{\frac{1}{2}}-5^{\frac{1}{2}}7^{\frac{3}{4}}}{9}.$$

11. If the roots of $x^2+px+q=0$ be in the ratio of 1 to 2, shew that one of them satisfies the equation

$$6px^2+(5p^2-6q)x+p(p^2-2q)=0.$$

Let a be one of the roots of the equation $x^2+px+q=0$, then $2a$ will be the other, and $a+2a=-p$; $\therefore$ $a=-\dfrac{p}{3}$. The second equation factors into $(3x+p)(2px+p^2-2q)=0$; $\therefore$ one root satisfying this equation is $x=-\dfrac{p}{3}=a$; $\therefore$ the two equations have one root $\left(-\dfrac{p}{3}\right)$ in common.