The left-hand side of the identity is

$$\frac{(a+b)^4\left\{(c+b)-(c+a)\right\}\left\{(c+b)+(c+a)\right\}+}{(a+b)^2\left\{(c+b)-(c+a)\right\}+}$$

anal. anal.

$$=\frac{(a+b)^4\left\{(c+b)^2-(c+a)^2\right\}+\text{anal.}}{(a+b)^2\left\{(c+b)-(c+a)\right\}+\text{anal.}}$$

$$=\frac{x^4(y^2-z^2)+y^4(z^2-x^2)+z^4(x^2-y^2)}{x^2(y-z)+y^2(z-x)+z^2(x-y)}$$

$$=\frac{(x^2-y^2)(y^2-z^2)(z^2-x^2)}{(x-y)(y-z)(z-x)}$$

$$=\frac{3(x^2-y^2)(y^2-z^2)(z^2-x^2)}{3(x-y)(y-z)(z-x)}$$

$$=\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$$

$$=\frac{\left\{(a+b)^2-(c+b)^2\right\}^3+\text{anal.}+\text{anal.}}{\left\{(a+b)-(b+c)\right\}^3+\text{anal.}+\text{anal.}}$$

$$=\frac{(a-c)^3\,(a+c+2b)^3+\text{anal.}+\text{anal.}}{(a-c)^3+\text{anal.}+\text{anal.}}.$$

NOTE.—It will be observed that x is put for $(a+b)$, y for $(b+c)$, and z for $(c+a)$.

109. If x be the circular measure of any arc, then

$$1-\frac{x^2}{\lfloor 2}+\frac{x^4}{\lfloor 4}-\ldots=\left\{1-\frac{2x^2}{\lfloor 2}+\frac{8x^4}{\lfloor 4}-\ldots\right\}^{\frac{1}{2}}$$

both sides being infinite.

$\because$ 2 $\cos^2 A=1+\cos 2A$,

$\therefore$ if x be the circular measure of the arc, measuring angle A, we have, by DeMoivre's Theorem, &c.,

$$2\left\{1-\frac{x^2}{\lfloor 2}+\frac{x^4}{\lfloor 4}-\frac{x^6}{\lfloor 6}+\ldots\text{ ad infin.}\right\}^2$$

$$=1+1-\frac{(2x)^2}{\lfloor 2}+\frac{(2x)^4}{\lfloor 4}-\frac{(2x)^6}{\lfloor 6}+\ldots$$

$$=2-\frac{4x^2}{\lfloor 2}+\frac{16x^4}{\lfloor 4}-\frac{64x^6}{\lfloor 6}+\ldots$$

$$\therefore\left\{1-\frac{x^2}{\lfloor 2}+\frac{x^4}{\lfloor 4}-\frac{x^6}{\lfloor 6}+\ldots\right\}^2$$

$$=1-\frac{2x^3}{\lfloor 2}+\frac{8x^4}{\lfloor 4}-\frac{32x^6}{\lfloor 6}+\ldots$$

or $$1-\frac{x^2}{\lfloor 2}+\frac{x^4}{\lfloor 4}-\frac{x^6}{\lfloor 6}+\ldots$$

$$=\left\{1-\frac{2x^2}{\lfloor 2}+\frac{8x^4}{\lfloor 4}-\frac{32x^6}{\lfloor 6}+\ldots\right\}^{\frac{1}{2}}$$

Solution by proposer, Prof. Edgar Frisby, M.A., N.O., Washington.

92. $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$; $\sin A=\dfrac{2\Delta}{bc}$;

$\cos B=\dfrac{c^2+a^2-b^2}{2ac}$; $\sin B=\dfrac{2\Delta}{ac}$;

$\cos C=\dfrac{a^2+b^2-c^2}{2ab}$; $\sin C=\dfrac{2\Delta}{ab}$;

$\cot A=\dfrac{b^2+c^2-a^2}{4\Delta}$; $\cot B=\dfrac{c^2+a^2-b^2}{4\Delta}$;

$\cot C=\dfrac{a^2+b^2-c^2}{4\Delta}$,

whence

$$\cot A+\cot B+\cot C=\frac{a^2+b^2+c^2}{4\Delta}, \quad (1)$$

$a^2\cot A+b^2\cot B+c^2\cot C$

$$=\frac{2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4}{4\Delta}$$

$$=\frac{16\Delta^2}{4\Delta}=4\Delta, \quad (2)$$

$$\cot^2 A=\frac{b^4+c^4+a^4+2b^2c^2-2a^2b^2-2a^2c^2}{16\Delta^2},$$

and similar quantities for B and C.

$\therefore \cot^2 A+\cot^2 B+\cot^2 C$

$$=\frac{3(a^4+b^4+c^4)-2(a^2b^2+a^2c^2+b^2c^2)}{16\Delta^2}$$

$\cot^2 A+\cot^2 B+\cot^2 C$

$$+\frac{2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4}{16\Delta}$$

$$=\frac{2(a^4+b^4+c^4)}{16\Delta^2}$$

$$1+\cot^2 A+\cot^2 B+\cot^2 C=\frac{a^4+b^4+c^4}{8\Delta^2}. \quad (3)$$

Many other results could be obtained by eliminating Δ, &c.

A different solution was sent in by S. T. G. Barton, Univ. Coll.