

from explanations and reasons, it is thought better to present the reasoning again with some fulness.

§ 56. Since $yv = n$, and y is not equal to n , y is the continued product of some of the factors of n , but not of them all. Let s, t , etc., be the odd factors of n of which y is a multiple; and b, d , etc., the odd factors of n of which y is not a multiple. Because $yv = n = b\beta$, and b is not a factor of y , b is a factor of v . Let $v = ab$; then $v\beta = an$. Therefore $F_{ev\beta} = F_0$. In like manner $X_{ev\beta} = X_0$, and so on as regards all those terms of the type $F_{ev\beta}$ in which $\frac{n}{\beta}$ or b is an odd factor of n , but not a factor of y . Hence, putting ev for z in the second of equations (108), and separating those factors of $R_{ev}^{\frac{1}{n}}$ that are of the type $F_{ev\beta}^{\frac{1}{n}}$ from those that are not,

$$R_{ev}^{\frac{1}{n}} = w'' A_{ev} (F_0^{\beta} X_0^{\beta} \dots)^{\frac{1}{n}} (P_{evm}^m \phi_{ev\sigma}^{\sigma} \dots)^{\frac{1}{n}},$$

w'' being an n^{th} root of unity. We understand that F_0^{β}, X_0^{β} , etc., are taken with the rational values which it has been proved that they admit, and, as in § 44, their continued product may be called Q . Then

$$R_{ev}^{\frac{1}{n}} = w'' A_{ev} Q (P_{evm}^m \phi_{ev\sigma}^{\sigma} \dots)^{\frac{1}{n}}. \quad (126)$$

When e is taken with the particular value c , let w'' become w^c , and when e has the value unity, let w'' become w^a . Then

$$\left. \begin{aligned} R_{cv}^{\frac{1}{n}} &= w^c A_{cv} Q (P_{cvm}^m \phi_{cv\sigma}^{\sigma} \dots)^{\frac{1}{n}} \\ R_v^{\frac{1}{n}} &= w^a A_v Q (P_{vm}^m \phi_{v\sigma}^{\sigma} \dots)^{\frac{1}{n}} \end{aligned} \right\} \quad (127)$$

Because R_1 is the fundamental element of the root of a pure uni-serial Abelian equation of the n^{th} degree, equations (3) and (5) subsist together; hence, because w^c is included in w^e ,

$$\left. \begin{aligned} (R_v R_1^{-v})^{\frac{1}{n}} &= k_1 \\ (R_{cv} R_c^{-v})^{\frac{1}{n}} &= k_c \end{aligned} \right\} \quad (128)$$

and where k_1 is a rational function of w , and k_c is what k_1 becomes by changing w into w^c . By putting e equal to unity in (109),

$$R_1^{\frac{1}{n}} = A_1 (P_m^m \phi_{\sigma}^{\sigma} \dots F_{\beta}^{\beta})^{\frac{1}{n}}.$$

Taking this in connection with the second of equations (127),

$$(R_v R_1^{-v})^{\frac{1}{n}} = w^a (A_v A_1^{-v}) Q (F_{\beta}^{-v\beta} \dots)^{\frac{1}{n}} \{ (P_{vm}^m P_m^{-vm}) (\phi_{v\sigma}^{\sigma} \phi_{\sigma}^{-v\sigma}) \dots \}^{\frac{1}{n}}. \quad (129)$$