The application of the rule here as in the earlier example furnishes an infinite series.

Referring to Ex. 4, p. 85, we see that we can find the sum of n terms of the series

$$1+2x+3x^2+\ldots\ldots$$

and *if x is numerically less than unity*, it can be shewn that the limit of the infinite series is $(1-x)^{-2}$.

Ex. 3. Find an expression in ascending powers of x for $(1+x)^{\frac{1}{2}}$.

We have $(1+x)^{\frac{1}{2}}=\sqrt{(1+x)}$. Let the square root be extracted,

$$\begin{array}{lll}
 & 1+x\Big(& 1+\frac{x}{2}-\frac{x^2}{8} \\
 & \underline{1} & \\
2+\frac{x}{2} & \quad +x & \\
 & \underline{+x+\frac{x^2}{4}} & \\
2+x-\frac{x^2}{8} & \quad -\frac{x^2}{4} & \\
 & \underline{-\frac{x^2}{4}-\frac{x^3}{8}+\frac{x^4}{64}} & \\
 & \quad\ldots\ldots &
\end{array}$$

If the binomial rule were applied we should obtain

$$1+\tfrac{1}{2}x+\frac{\frac{1}{2}(\frac{1}{2}-1)}{1.2}x^2+\ldots\ldots$$

or

$$1+\frac{x}{2}-\frac{x^2}{8}+\ldots\ldots$$

a result which agrees with the value found for $(1+x)^{\frac{1}{2}}$. It is plain that the series will not terminate, and it could be shewn that the infinite series has a meaning only *when x is numerically less than unity.*

It is also to be noted that $(1+x)^{\frac{1}{2}}$ has two values. In finding the square root we might have started with -1 as well as with $+1$ and the two values found would differ only in sign. The binomial rule gives only one of the roots.

A study of the preceding examples would seem to lead to the conclusion that the Binomial Theorem is valid, under certain restric-