$$v=\frac{Q}{A}=C(rs)^{0.5}\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The best size of conduit is to be determined for a known value of the flow Q; that is, in Equation (4) Q is to be taken as constant. In this case,

$$\frac{dQ}{ds}=0$$

$$\frac{dA}{ds}=-\frac{2A}{5s};\quad \frac{dW}{ds}=-\frac{1w}{5s};\quad \frac{dr}{ds}=-\frac{1r}{5s}\ldots\ldots(5)$$

With Q constant, a change in s can be offset by a change in either A or r, or in both; that is, either the size or the shape of the conduit can be varied to keep Q constant.

There is no way of expressing a general relation between A and r, but for any chosen shape, as, for example, a rectangle or semi-circle, the area is proportional to the square of any linear dimension; that is,

$$A=ar^2=wr=\frac{w^2}{a}\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

The value of a, of course, varies, but for usual forms the differences are not great, and the influence of changes in a on the economical section is slight; in fact, it can be shown that for the best section

$$A\sim a^{1/7}$$

for the conditions of Equation (7a). Table I. gives values for the conditions of Equation (7a). The accompanying

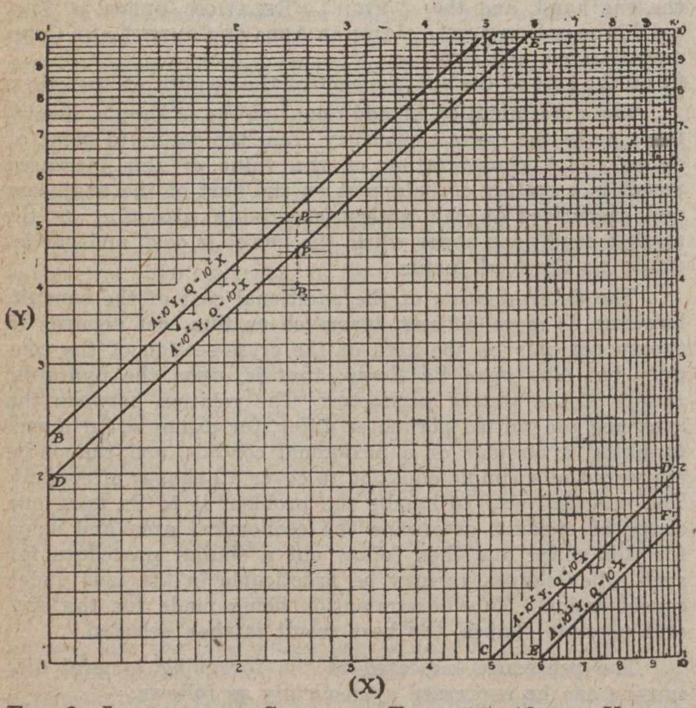

FIG. 2—LOGARITHMIC GRAPH OF EQUATION 18 FOR VALUES OF Q FROM 100 TO 10,000 SEC. FT.

table gives value of a for the usual shapes. For preliminary calculations a=9 may be used.

The cost of a water conduit can be expressed as a constant, representing the cost of a large part of the preliminary work and plant, plus an amount depending on the size and surface area. In general, the cost per foot may be expressed by

$$D=k_0+kA^n\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

where D=cost per foot, dollars

k_0=constant part of cost per foot

k = a constant

n = an exponent whose value lies between 1 and 0.5.

In any specific case, when all the conditions are known, estimates of the total cost per foot of the conduit should be made for three or more different cross-sections; plotting these values will enable both k and n to be determined.

For example, let Fig. 1 represent the cost per foot for a certain conduit; then k_0 is given at once by the curve, and from

$$\frac{D_2-k_0}{D_1-k_0}=\left(\frac{A_2}{A_1}\right)^n$$

n is determined. Each pair of points should be used and the value of n found. The values of n and k_0 being known, k may be obtained from

$$k=\frac{D_1-k_0}{A^n}.$$

Other methods could be used. The gist of the matter is that the accurate way is to make detailed estimates for several cross-sections and determine the constants from an analysis of these estimates.

Two extreme cases simplify the formula: First, when the increment cost is proportional to the area, as in a heavy rock cut, then

$$D=k_0+kA\ldots\ldots\ldots\ldots\ldots\ldots(7a)$$

and, second, where the increment cost is proportional to the surface, or the wetted perimeter, as for a flume, then

$$D=k_0+kA^{0.5}\ldots\ldots\ldots\ldots\ldots\ldots(7b)$$

These are considered later.

If i represents the total rate of returns expected on all expenditures on the property, including interest, amortization and profit, then

$$I=iL\,(k_0+kA^n)\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

gives the total returns from this investment, and a change ds in s calls for a change in returns of

$$dI=niLkA^{n-1}\frac{dA}{ds}\,ds\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

or, from Equation (5)

$$dI=-\frac{2niLkA^n}{5s}\,ds\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

This saving, due to an increase in s, must be at least equal in value to the power lost, and indeed should exceed it by some margin; this margin can be included in the overall rate of return i, and therefore

$$dI=mdp\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Substituting in (11) from (10) and (3), there results

$$5meQs=2kniA^n$$

Substituting further from (6) and (4), namely,

$$s=\frac{v^2}{C^2r}=\frac{v^2}{C^2(A/a)^{0.5}},\text{ and } Q=Av,$$

giving finally

$$A^{n-0.5}=\frac{2.5mea^{0.5}}{nikC^2}v^3\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

This may also be written

$$A^{(n+2.5)}=\frac{2.5mea^{0.5}}{nikC^2}Q^3\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

If

$$N=\frac{2.5mea^{0.5}}{nikC^2}\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

then

$$A^{(n+2.5)}=NQ^3\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

The best way to handle this equation for engineers is by logarithmic plotting. From (15)

$$\log A=\frac{\log N}{(n+2.5)}+\frac{3}{(n+2.5)}\log Q\ldots\ldots\ldots\ldots(16)$$

When n is known, this can be readily plotted for any range of Q desired. As an illustration, assume:—

$$m=\$10,\ e=0.67\times0.085=0.057$$
$$i=0.15,\quad C=120$$

then

$$N=\frac{10^{-3}\times2}{nk}$$

If, further, n=0.75 and k=\$0.10,

$$n=10^{-2}\times2.67$$

and

$$\log A=\frac{\log 10^{-2}\times2.67}{3.25}+0.925\log Q$$
$$=-0.485+0.925\log Q\ldots\ldots\ldots\ldots(18)$$

(Concluded on page 559)