

in question, when the origin of its principles, the method of procedure, and its highest developments, are comprehended in two words *abstraction* and *generalization*. It may be said that mathematical abstraction and generalization, are different from those of common observation and experience, and therefore they do not cultivate the higher intellectual faculties. We admit that mathematical processes evolve truths characterized by an immutability that can never distinguish the generalizations of the world of probabilities. But the *mental powers that abstract and generalize are the same*, whether directed to the investigation of *necessary* or *contingent* matter, and they surely cannot be less efficiently cultivated in the one case in consequence of their attaining clear and certain results, than in the other, because they are limited by probability.

3. But the study of Mathematics is of great value as correcting the vice of mental distraction, and forming the opposite habit of continuity of thought.

The importance of early forming the habit of deliberate concentration, can hardly be too highly estimated. In the commencement of all intellectual effort there is difficulty, and the success attending such effort must be in proportion to the power of continuous thought we are capable of exercising.

For, when we first direct our attention to subjects of investigation, our minds are distracted by numerous extraneous thoughts arising from the manifold circumstances that surround us, and we are thus prevented from bringing our intellectual energies fully to bear upon the subject. Nor is the difficulty overcome by a simple exertion of will. Even when we feel the necessity of concentrating power for the accomplishment of our object, and determine to exclude all distracting elements, a thousand obtrusive notions will spring up to confuse our thoughts and dissipate our energies; and for a long time even after the subject of investigation begins to allure us by presenting interesting thoughts for our contemplation, the glimmerings and associations of preceding reflections will obtrude themselves and prevent the full and free exercise of our intellectual powers. When opposed by these obtrusive elements, is it possible for the mind to energize freely and successfully prosecute difficult investigations? Can it reach its highest attainable success when its operations are impeded by distracting thoughts, and its energies dissipated by efforts to concentrate its powers upon the object in view? On the contrary, the *habit of concentration* must be acquired before the mind can energize with the probability of attaining the highest success within its reach. But this can be accomplished only by time and frequent practice in the effort. The first effort at concentration is attended with great difficulty; but power is gained by repeated effort; every succeeding effort will become less difficult than the preceding; the elements of distraction gradually diminish in frequency and power, and at last the mind can almost involuntarily concentrate its energies upon its object, and attain its highest pleasure and most certain results, in the unimpeded operation of its powers. So important is this power of continuity of thought, that some have considered it identical with genius, while all admit that it is an invariable concomitant of genius, and a necessary condition of its greatest achievements. Sir Isaac Newton, with his usual modesty, attributed his success, not to the force of superior genius, but simply to his power of patient attention. "Genius," says a celebrated philosopher, "is only a protracted patience." Granting that there may be great powers of mind without a corresponding power of concentration, they cannot accomplish great results; but on the other hand, mediocrity accompanied by a high degree of this power may be elevated almost to the rank of genius.

Now, in Mathematics, we are accustomed to emancipate ourselves from the disturbing influences that surround us. We *must* concentrate attention on the truths bearing on the investigations, or there is no possibility of reaching a successful issue. The mind is required to keep constantly before it a vast number of already established premises, and to exercise its powers in the selection and application of those specially adapted to the end in view. And as the connection between the successive steps is, in general, difficult to grasp, in consequence of the abstract nature of the conceptions involved, the continuity of thought necessarily required must be both intensive and extensive,—intensive, as requiring a high degree of mental power to grasp the conceptions and their relations; extensive, as involving the contemplation and analysis of a large number of distinct, though related truths. But the fact that a high degree of abstractive power is required—as already shown—in both elementary and advanced Mathematics, proves conclusively their utility in cultivating the power of continuous thought. I may

remark, in preparing the mind for the careful contemplation of its own transient phenomena. The habit of concentration, to which I have referred, can be formed only by acts of "patient attention" which gradually increase in difficulty. This is accomplished by the study of Mathematics. The elementary propositions demand a certain amount of concentrative power, and every successive step demands a somewhat higher degree in consequence of the greater difficulty and greater number of the terms involved, and from this constant repetition and exercise, commencing with the simpler acts and rising gradually to the more profound, the power of continuous thought is increased to a higher degree than can be reached by any other course of discipline. If it be said that Mathematics, from the very fact that they demand a degree of continuity of thought in their most elementary propositions, are thus unfitted to remedy the vice of distraction, we reply that if there be any possible corrective discipline, it must be such as begins with the simpler efforts of the power of continuity, and gradually proceeds to the more difficult and prolonged; and the mind that cannot exert concentration sufficient to master the elementary propositions of Mathematics, though it may sometimes give evidence of latent power, will never be likely to attain by any other means this invaluable habit of mind.

4. The study of Mathematics develops the *power of observation*, and cultivates the *imagination*, whether considered as a representative or a creative power. By observation we mean the power of fixing attention on material or mental objects so as to note their distinctive properties, and their points of resemblance and of difference. From the most elementary Mathematical notions to the highest range of Mathematical investigation, this power is exercised. Even the first notions of number and of form which underlie Arithmetic and pure Geometry, are acquired by observation. Nor is it alone in securing the fundamental notions of number and form that observation plays so important a part. "The very genius of common Geometry is that it is but a series of observations. The figure being before the eye in actual representation or before the mind in conception is so closely scrutinized that all its distinctive features are perceived, auxiliary lines are drawn (the imagination leading in this) and a new series of inspections is made; and thus aided by direct simple observations, the investigation proceeds. So necessary is observation in Geometry that Comte, the ablest writer on the philosophy of Mathematics, is disposed to class Geometry—in view of its methods—with the natural sciences, as being based on observation. When we consider applied Mathematics, we have only to notice that the exercise of this faculty is so essential that the basis of investigation, the very materials with which we build, have received the name *observations*.

Further, the *representative power of imagination*, as constantly exercised in presenting to the mind intuitions of space, and the complicated relations of external things, must be greatly strengthened and developed; and the beauty, order, and harmony disclosed in terrestrial phenomena, and in the starry regions where worlds on worlds arise, must permit abundant materials for the exercise of its creative powers. And this creative faculty has constant exercise in all original Mathematical investigation from the solution of the simplest problem to the discovery of the most recondite truths; for it is not by intuitive, consecutive steps that we advance from the known to the unknown, the imagination, rather than the logical faculty, leads in this advance. In fact, practical observation is often in advance of logical exposition. In the discovery of truth, the imagination habitually presents hypothesis, and observation supplies facts which it may take ages to connect logically with the known. That the imagination, and not the logical faculty, leads in all original investigations will be admitted by any student who has ever succeeded in producing an original demonstration of even one of the simple propositions in Geometry.

III.

Let us now proceed to notice the VALUE OF MATHEMATICS AS A LOGICAL EXERCISE OF MIND.

1. It habituates the mind to the use of correct forms of reasoning.

However valuable pure logic may be as the science of the laws of thought, its highest utility is realized only in the practical application of its principles. The bare study of the formal laws of thought cannot exercise the mind in the forms which it must follow in all sound reasoning. Logic may unfold the characteristics of these laws, and thence deduce the necessary conditions of cogency in reasoning. But the mind can acquire the