11. Find the condition that the equations

$$\left.\begin{array}{l} ax^2+bx+c=0 \\ a'x^2+b'x+c'=0 \end{array}\right\}$$

may have (1) one root in common; (2) both roots in common.

(1) The roots of the given equations are

$$\frac{-b \pm \surd(b^2-4ac)}{2a}, \text{ and } \frac{-b' \pm \surd(b'^2-4a'c')}{2a'}.$$

If the equations are to have one root common we must have

$$\frac{-b \pm \surd(b^2-4ac}{2a} = \frac{-b' \pm \surd(b'^2-4a'c')}{2a'},$$

which, after transposing, squaring, etc., gives

$$(ac'-a'c)^2 - bb'(ac'-a'c) + b^2c'a' + b'^2ca = 0.$$

The following is the more usual solution of this question:—

Suppose that x is the common root, then x satisfies both equations, multiplying the first c' and the second by c, and subtracting we have

$$(ac'-a'c)x^2 + (bc'-b'c)x = 0,$$

or $$(ac'-a'c)x = (b'c-bc') \ldots\ldots (1)$$

Again, multlplying the first by a' and the second by a, and subtracting, we have

$$(a'b-ab')x = ac'-a'c \ldots\ldots (2).$$

Dividing (1) by (2), and simplifying, we have

$$(ac'-a'c)^2 = (a'b-ab')(b'c-bc').$$

(2) If the equations are to have both roots common, we must have the sum and product of the roots of the first equation respectively equal to the sum and product of the roots of the second equation; now the

sum of roots in the first equation $= -\frac{b}{a}$, product $= \frac{c}{a}$

" " " second " $-\frac{b'}{a'}$ " $= \frac{c'}{a'}$;

$$\therefore \frac{b}{a} = \frac{b'}{a'}, \text{ and } \frac{c}{a} = \frac{c'}{a'},$$

which may be written symmetrically

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'},$$

thus showing that the second equation must be deducible from the first.