# SCHOOL WORK.

## MATHEMATICS.

ARCHIBALD MACMURCHY, M.A., TORONTO.
EDITOR.

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### ANSWERS TO PROBLEMS.

By Prof. Edgar Frisby, of the Naval Observatory, Washington, D.C.

81. Solve, by a simple quadratic method, the equation $x^6+12x^5+14x^4-140x^3+69x^2+128x-84=0$.

81. $x^6-12x^5+14x^4-140x^3+69x^2+128x-84=0$, is immediately seen by inspection to be satisfied by $x=+1$ and $x=-1$, it is therefore divisible by $x^2-1$, and becomes $(x^2-1)(x^4+12x^3+15x^2-128x+84)=0$, the right hand factor is equal to $x^4+12x^3+36x^2-20(x^2+6x)-x^2+8x+84=0$, or $(x^2+6x)^2-20(x^2+6x)+100=x^2+8x+16$,

$$x^2+6x-10=\pm(x+4)$$
$$x^2+5x-14=0$$
$$x^2+7x-6=0$$

the first gives $x=\dfrac{-5\pm\sqrt{81}}{2}=-7$ or $+2$

the second $x=\dfrac{-7\pm\sqrt{73}}{2}$,

the 6 roots are therefore: $1$, $-1$, $2$, $-7$, $\dfrac{-7+\sqrt{73}}{2}$, and $\dfrac{-7-\sqrt{73}}{2}$.

85. Construct a triangle, the product of two sides, the medial line to the third side, and the difference of the angles adjacent to the third side, being given.

85. $$\frac{a+b}{c}=\frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}} \quad (1)$$

$$\frac{a-b}{c}=\frac{\sin\frac{A-B}{2}}{\cos\frac{C}{2}} \quad (2)$$

$$a^2+b^2=\frac{C^2}{2}+2d^2 \quad (3)$$ $d$ being the medial line.

From (1) $(a^2+b^2+2ab)$

$$\sin^2\frac{C}{2}=c^2\cos^2\frac{A-B}{2} \quad (4)$$

from (2) $(a^2+b^2-2ab)$

$$\cos^2\frac{C}{2}=c^2\sin^2\frac{A-B}{2} \quad (5)$$

adding and subtracting these two equations we have

$$a^2+b^2-2ab\cos C=c^2 \quad (6)$$
$$(a^2+b^2)\cos C-2ab+c^2\cos(A-B)=0 \quad (7)$$

elim. $\cos C$

$$(a^2+b^2)(a^2+b^2-c^2)-4a^2b^2+2abc^2\cos(A-B)=0 \quad (8)$$

substituting from (3)

$$\left(2d^2+\frac{c^2}{2}\right)\left(2d^2-\frac{c^2}{2}\right)-4a^2b^2+2abc^2\cos(A-B)=0 \quad (9)$$

whence

$$c^4-8ab\cos(A-B)c^2+16a^2b^2\cos^2(A-B)=16(d^2-a^2b^2\sin^2\overline{A-B})$$

or $C^2=4(ab\cos\overline{A-B}\pm a)$

where $d^2-a^2b^2\sin^2\overline{A-B}=a^2$

and $c$ is therefore known.

$a$ and $b$ can be obtained from (3)

$$a=\frac{1}{2}\sqrt{\frac{c^2}{2}+2d^2+2ab}+\frac{1}{2}\sqrt{\frac{c^2}{2}+2d^2-2ab}$$

$$b=\frac{1}{2}\sqrt{\frac{c^2}{2}+2d^2+2ab}-\frac{1}{2}\sqrt{\frac{c^2}{2}+2d^2-2ab}$$

$C$ from either (1), (2), (6) or (7), and

$$A=90+\frac{A-B-C}{2}$$

$$B=90-\frac{A-B+C}{2}.$$

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## MODERN LANGUAGES.

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### EXERCISES IN ENGLISH.

CONTRIBUTED BY MR. W. H. JOHNSTON, HAY P. O.

1. Change to indirect narrative:

(*a*) "Zotoff," she murmured, "goodbye, I am dying."