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given angle; and that the triangle LGG_1 shall differ from the triangle LGF by a space less than any given space, the sum of its angles at the same time differing from the sum of the angles of the triangle LGF by an angle less than any given angle; and so on, till the whole of the triangle LBD has been exhausted, except a remainder LBE, which is less than the triangle to which it is adjacent. Proceed next to divide the triangle LDC into triangles LDT, LTH, &c., related to the triangle LFD and to one another in the same manner as the triangles LFG, LGG₁, &c.; the remainder LMC being finally left over, less than the triangle to which it is adjacent. Then, since any two adjacent triangles in the series,

LDF, LFG, LGG₁, &c., (1)

which together constitute the triangle LDE, may be made as nearly equal as we please, we can make every one of them as usarly equal to the first as we please. And, from a similar consideration, it appears that we can at the same time make the sum of the angles of any triangle in the series as nearly equal as we please to the sum of the angles of the first. In like manner we can make every one of the triangles in the series,

which together constitute the triangle LDM, as nearly equal to LDF as we please; the sum of the angles of each being at the same time made as nearly equal as we please to the sum of the angles of the triangle LDF. Let there be N terms in the series (1), and n in the series (2). Then

 $LED = N \text{ times } LFD \Leftrightarrow Q; \qquad (3)$

Q being a quantity which we may arrange to have as small as we please. In like manner,

 $LMD = n \text{ times } LFD \Leftrightarrow q; \qquad (4)$

q being a quantity which we may arrange to have as small as we please. Again, if S_1 be the sum of the angles for the triangle LFD, $S_1 \leq h_1$ the sum of the angles of the triangle LFG, $S_1 \leq h_2$ the sum of the angles of the triangle LGG₁, and so on, and S_2 the sum of the angles of the triangle LED, we have

$$S_{2} = NS_{1} - 2(N-1) \Leftrightarrow h_{1} \Leftrightarrow h_{2} \And \&c.$$

$$\therefore 2 - S_{2} = N(2 - S_{1}) \Leftrightarrow h; \qquad (5)$$