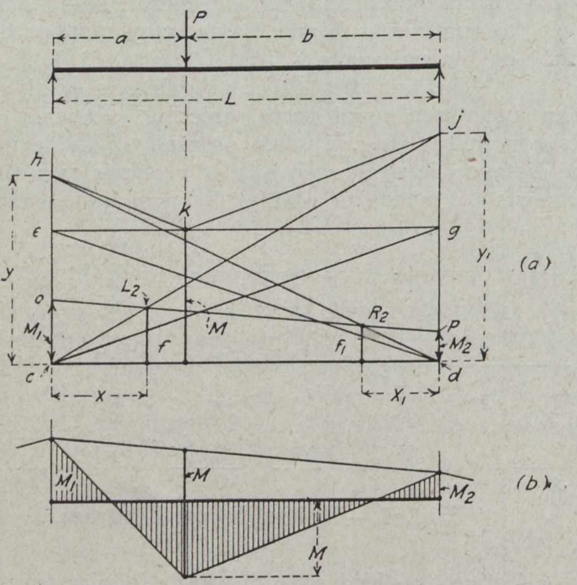
To locate the inflection point L_3 for span L_3, start again, from the inflection point L_2 just found, and draw an arbitrary line (4), proceeding as before.

The inflection points R_1 and R_2 are obtained in the same manner, except that it becomes necessary to start from the point h, advancing to a. It is clear, therefore, that the designations regarding the inflection points are essentially the same for either L or R inflection points, requiring no further detailed explanation. A brief demonstration of the various steps to be taken in determining the bending moments for an individual span carrying a uniform or concentrated load, will be given.



$$M = P\,\frac{a(L-a)}{L}$$

$$\text{Proof}-\; y = M\cdot\frac{L+a}{L} \qquad y' = M\cdot\frac{2L-a}{L} \qquad f = y'\frac{x}{L} = M\cdot\frac{2L-a}{L}\cdot\frac{x}{L}$$

$$f_1 = y\cdot\frac{X_1}{L} = M\cdot\frac{L-a}{L}\cdot\frac{X_1}{L} \qquad M_1 = f + (f-f_1)\cdot\frac{x}{L-x-X_1} =$$

$$M\cdot\frac{2L-a}{L}\cdot\frac{x}{L} + M\cdot\left(\frac{2L-a}{L}\cdot\frac{x}{L} - \frac{L+x}{L}\cdot\frac{X_1}{L}\right)\cdot\frac{X_1}{L-x-X_1} =$$

$$M\cdot\frac{x}{L}\cdot\frac{2L-a-3X_1}{L-x-X_1} = P\cdot\frac{a(L-a)}{L}\cdot\frac{x}{L}\cdot\frac{2L-a-3X_1}{L-x-X_1}$$

$$M_2 = P\cdot\frac{a(L-a)}{L}\cdot\frac{X_1}{L}\cdot\frac{L+a-3X}{L-X-X_1}$$

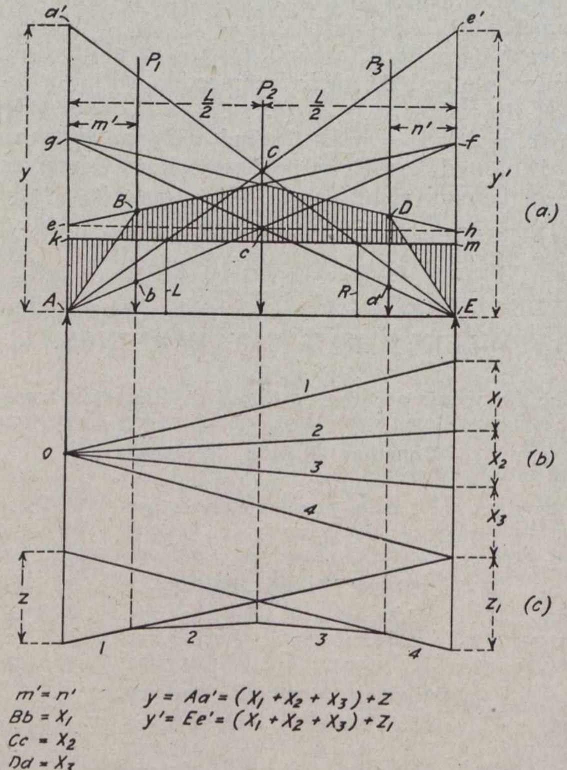
Fig. 3.—Diagram for Concentrated Load.

Uniform Loading.—The method of obtaining the bending moments for span L_2 uniformly loaded is illustrated in Fig. 2. Compute, then, plot to any convenient scale, the moment, M at $\frac{L}{2}$, as if the beam was discontinuous, and draw lines ab and bc. Where these lines cut their respective verticals L_2 and R_2, two points are determined, enabling the bending moments M_1 and M_2 at the intermediate supports, to be deduced, by drawing the line de through these points as shown in (a). It is then a simple matter to construct the bending moment diagram for the entire span, requiring the construction of the parabola as in (b).

Concentrated Loads.—Referring to the same span L_2, as discontinuous, the effect of a concentrated load P, will now be taken into consideration (Fig. 3), M in this case being used to indicate the bending moment directly at the point of application of the load. Having plotted the value of M in (a), draw eg parallel to the base cd. Join cg and ed, and then draw hk parallel to ed and kJ parallel to cg. Join cJ and dh and where these lines cut their respective verticals L_2 and R_2, draw the line op. The negative bending moments M_1 and M_2 can then be scaled off, and the bending moment diagram (b) constructed.

Irregular Load Systems.—In Fig. 4, a system of concentrated or superimposed loads, P_1, P_2 and P_3 is considered. In order to apply the graphical method to this case, the bending moment diagram $ABCDE$ for the span, irrespective of the beams being continuous, is plotted. After this has been done, a special construction is required in order to find the positive and negative moments due to the continuity of the beam. The method is as follows: Produce line BC in both directions, until it intersects the continuation lines of the supports at e and f. By a similar reasoning, using line CD, points g and h are determined. Now connect g and E, A, and f and E and h by means of straight lines. These operations serve for determining the three points b, c, and d. Symbolizing the distances Bb, Cc, and Dd by X_1, X_2 and X_3, respectively, plot them along the line of one of the supports and with a pole distance equal to the span, draw the polar diagram (b) from which diagram (c) is constructed. Now lay off y and y' as shown in the figure and draw line $a'E$ and $A'e$. Where these lines cut the respective lines produced from the inflection points L and R, the necessary two points are obtained to determine the direction of line km, completing the diagram.

So far the bending moments for only one span have been shown diagrammatically. The moments at the supports of a continuous beam have been deduced in Figs. 5 and 6 and can be determined easily. The simplest procedure for obtaining these moments is to derive the moments for the loaded span, by the means explained above, the second element requiring recognition of the remaining inflection points, which governed the direction



$m' = n'$ $\quad y = Aa' = (X_1 + X_2 + X_3) + Z$

$Bb = X_1$ $\quad y' = Ee' = (X_1 + X_2 + X_3) + Z_1$

$Cc = X_2$

$Dd = X_3$

Fig. 4.—Diagram for Irregular Loads.

of the lines instrumental in obtaining these moments. To simplify explanations, the arbitrary term, instrumental line, will be introduced for want of better terminology, there being no standard technical term that can be used to designate this kind of a line. Considering the span just to the left of the loaded span, from the point of greatest bending moment (a), previously found, the instrumental line is drawn through the inflection point L_3,