FORMULAE FOR REINFORCED CONCRETE IN FLEXURE IN THE LIGHT OF EXPERIMENTAL DATA.*

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Proposition 1.—A plane section before bending becomes after bending a curved section through which an imaginary plane passes and touches three principal parallel lines in the curved section, viz.,—a line in the plane of the top fibres of the concrete; a line in the plane of the centre of gravity of the areas of the steel reinforcement; and a line in the plane of the neutral axis. (In plain beams of wood, steel or concrete, the curved section may possibly take the form of the dotted line "D" in Figure 1 on the tension side and the imaginary section would then touch the plane of the bottom fibres of the concrete instead of "C").

Formulae based upon correct theoretical assumptions point out theoretical values that are supported by experimental data. This is not the case with the formulae now used for reinforced concrete in flexure because there are anomalies existing between theory and practice. It was the writer's original intention to present the records of experimental data

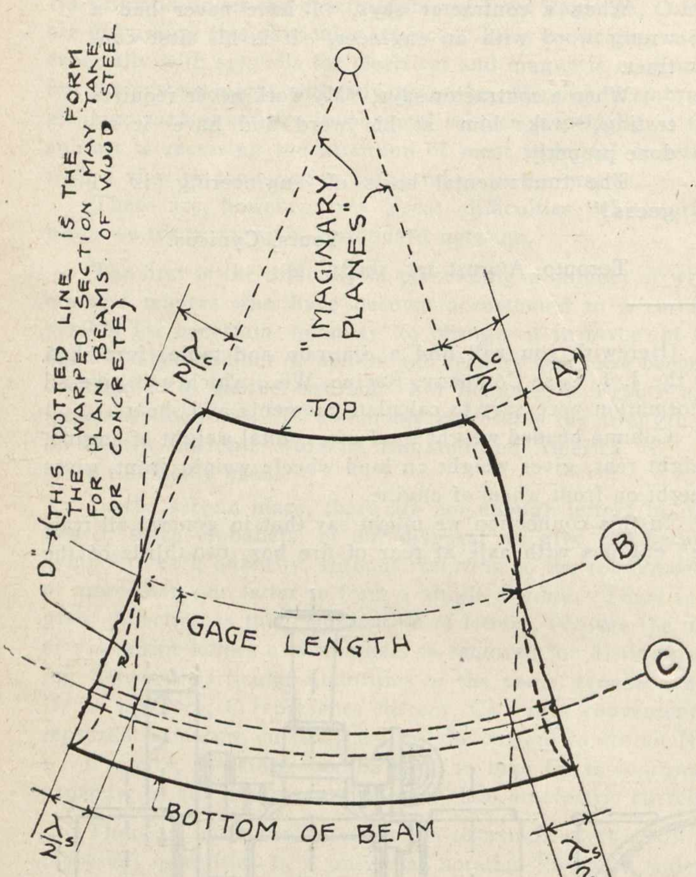

Fig. 1, Showing a Section of Concrete Beam After Bending.

that tend to account for several of these anomalies, but owing to the importance of the question, whether a plane section before bending remains a plane section after bending, it was decided to only present this one phase of the subject in order to abbreviate the paper and thus provide more time for the discussion of it by the members of this Convention. All other facts in connection with this subject are therefore omitted and will probably be given at some future date to some of the technical papers for publication.

There are two theories upon which these formulae are based. In one, the theory of **Straight Line** stress distribution, it is assumed that the modulus of elasticity of the concrete in compression is constant throughout the working limits of stress, and that a plane section before bending remains a plane section after bending; in the other, the **Parabolic** theory of stress distribution, it is assumed that there is a clearly defined decrease in the modulus of elasticity of the concrete, and also, as in the first case, that a plane section before bending remains a plane section after bending. A number of other assumptions are also made, but this paper will be limited to the consideration of the foregoing.

In considering the modulus of elasticity of concrete in compression, it was found that the curves plotted from the results of the experiments conducted under the direction of Professor Bach of Stuttgart University, and those conducted under the direction of Professor Talbot, of the University of Illinois, practically coincided within the working limits of stress,—that is, for stresses from one-fourth of the ultimate strength to the ultimate strength of the concrete in compression, a range which practically covers the field of experimental investigation.

In Bulletin No. 14 of the University of Illinois Engineering Experiment Station, Professor Talbot has this to say on the question of the stress-strain relation:—"Concrete does not possess the property of proportionality of stress and deformation for wide ranges of stress as does steel; in other words, the deformation produced by a load is not proportional to the compressive stress. . . . . Various curves have been proposed to represent the stress-deformation relation but the parabola is the most satisfactory general representation. Frequently the parabola expresses the relation almost exactly." In the light of these facts which are practically supported by the experiments of Professor Bach, we are led to believe that the curve of the modulus of elasticity of concrete in compression follows the law of the parabola—the rate of decrease

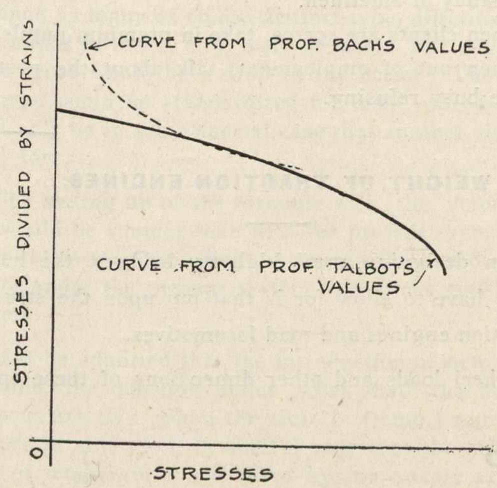

Diagram No. 1.

being more rapid for stresses near the ultimate strength than for the low stresses. In other words, the previously-mentioned assumption, that there is a clearly defined decrease in the modulus of elasticity of the concrete, is supported by experimental data. (For stresses under one-fourth of the ultimate strength, there is a difference existing between the results of these eminent investigators. The two curves are herewith reproduced to the same scale.—See Diagram No. 1, and note the upward tendency of the curve plotted from the results of Professor Bach's experiments on low stresses. However, since stresses below one-fourth can only have a nominal influence on the results of our investigation, we are justified in tentatively accepting Professor Talbot's parabolic curve as given, i.e., from zero stress to the ultimate strength). It is not proposed to accept this curve as representing the actual values of the modulus of elasticity, but rather, as representing the nature of the change that takes place in the modulus of every specimen of concrete in compression as the stress uniformly increases from low stresses to the ultimate stren . In other words, we are concerned, in this discussion, with the law underlying the stress-strain relation, rather than the actual stress-strain records. In accordance with this assumption, the curve in Diagram No. 2 has been made to conform to the curve suggested by Professor Talbot, as representing the changes in the stress-strain relation throughout the whole range of compressive stresses. The abscissas in Diagram No. 2 represent the stresses in percentages of the total stress that

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