this result that in general $\tan \frac{1}{2}A = \frac{1-\cos A}{\sin A}$? If not, why not?

x. Establish the identity

$$\tan \frac{x+y}{2} \tan \frac{x-y}{2} = \frac{\operatorname{cosec} 2x \operatorname{cosec} y - \operatorname{cosec} 2y \operatorname{cosec} x}{\operatorname{cosec} 2x \operatorname{cosec} y + \operatorname{cosec} 2y \operatorname{cosec} x}.$$

Shew that if $\cot \frac{1}{2}\alpha + \cot \frac{1}{2}\beta = 2 \cot \theta$, then

$$\{1 - 2 \sec \theta \cos (\alpha - \theta) + \sec^2 \theta\}$$
$$\{1 - 2 \sec \theta \cos (\beta - \theta) + \sec^2 \theta\} = \tan^4 \theta.$$

xi. In a triangle ABC, prove that with the usual notation

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c},$$

and $\quad a^2 = b^2 + c^2 - 2bc \cos A.$

Shew also that

$$a^2 \cos 2(B-C) = b^2 \cos 2B + c^2 \cos 2C + 2bc \cos (B-C).$$

xii. Find expressions for the radii of the inscribed and circumscribed circles of a triangle ABC, and shew that the ratio of the former to the latter is

$$4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}.$$

If D, E, F are the points of contact of the inscribed circle with the sides BC, CA, AB respectively, shew that if the squares of AD, BE, CF are in arithmetical progression, then the sides of the triangle are in harmonical progression.

PROBLEM.

Let ABC be an equilateral triangle and O any point within it, let the perpendiculars OD, OE, OF be drawn; it is required to find the position of O and the length of a ladder that will reach from O to A, B or C.

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SOLUTIONS

Of problems for the "All the years," University of Toronto, 1880, by Angus MacMurchy.

2. If an exterior angle of a triangle be bisected by a straight line which likewise cuts the base; the rectangle contained by the sides of the triangle, together with the square on the line bisecting the angle is equal to the rectangle contained by the segments of the base.

Let ABC be the triangle, let the line bisecting the exterior angle meet BC produced in D, about ABC describe a circle, let DA produced meet this circle in E. From similar triangles EAC, BAD, $EA.AD = BA.AC$,

$$\therefore EA.AD + AD^2 = BA.AC + AD^2$$
$$ED.DA = BD.DC = BA.AC + AD^2.$$

3. If x, y, z be the perpendiculars from the angles of a triangle on the opposite sides, and if

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2}{\sigma},$$

prove that

$$4\sqrt{\frac{1}{\sigma}\left(\frac{1}{\sigma}-\frac{1}{x}\right)\left(\frac{1}{\sigma}-\frac{1}{y}\right)\left(\frac{1}{\sigma}-\frac{1}{z}\right)} = \frac{1}{\text{area of triangle}}.$$

$$ax = by = cz = 2S,$$
$$\therefore \frac{2}{\sigma} = \frac{s}{S}.$$

Substitute, and

$$\sqrt{\frac{s(s-a)(s-b)(s-c)}{S^4}} = \frac{1}{S}.$$

IV. Prove that every power of the sum of two squares may be divided into two parts, each of which is the square of an integer.

Let

$$(a+x\sqrt{-1})^n = a_0 + a_1\sqrt{-1} - a_2 - a_3\sqrt{-1} + \ldots$$
$$(a-x\sqrt{-1})^n = a_0 - a_1\sqrt{-1} - a_2 + a_3\sqrt{-1} + \ldots$$
$$\therefore (a_0 - a_2 + a_4 - \ldots)^2 + (a_1 - a_3 + a_5 - \ldots)^2 = (a^2 + x^2)^n.$$

V. Find the sum of the series

$$\frac{4}{1 \cdot 5} + \frac{9}{5 \cdot 14} + \frac{16}{14 \cdot 30} + \frac{25}{30 \cdot 55} + \ldots \text{to } n \text{ terms},$$

the last factor in the denominator being the sum of the other factor and the numerator.

$$\text{Let } S = \frac{2^2}{1^2(1^2+2^2)} + \frac{3^2}{(1^2+2^2)(1^2+2^2+3^2)} + \ldots + \frac{(n+1)^2}{\Sigma n^2 . \Sigma \overline{n+1}^2}.$$