the killing answer is, "Oh what bosh!" We have not space to give the professor's other and equally significant examples. The inferences —some of them formally stated—which he draws from his "definition" and examples are:

(1) In division the divisor and the dividend always have the "same name." The quotient is concrete.

(2) In division the quotient *always* equals the *dividend.*

(3) The divisor cannot be greater than the dividend—"8 ÷ 8 how absurd."

(4) The divisor can *never* be an *abstract* number.

(5) Finding the equal parts of a number "is *not* division; but differs widely from it."

Now a sufficient answer to all this is supplied by the professor himself: "Oh what bosh!" We not only call it bosh; we shall prove it bosh—so far at any rate as what is self-evident is capable of proof.

(1) Had the writer borne in mind the principle stated in his introduction, viz.: that in finding the volume of anything we are *simply repeating a number of units a certain number of times*—recalling, further, the well known fact that the operation of division is the inverse of that of multiplication, he would not have been found wallowing in a slough of absurdities. The question, How often is \$4 contained in \$12, is the inverse of the question, What is the amount of \$4 repeated three times? The operation in this case is \$4 + \$4 + \$4 = \$12; or, using the multiplication table, which is but a series of *remembered addition results*—\$4 × 3 = \$12; where clearly the *three* denotes *how many* addends there are—how many groups of *four* things each—and is therefore purely a number—i.e. an "abstract" number; for the conception would *not change with any change of addend.* This concept, THREE, would remain absolutely unchanged if the *groups of things* were changed indefinitely either in number or in kind—i.e. we might have groups of 1 or 2 or 3 or 4. . . . or *n* things each and the *things* might be dollars, or apples, or any thing else in the universe of things.

No other meaning for the *multiplier* can be conceived by a mathematically sane mind. How then is the *inverse* problem connected with this? In multiplication we have the group of things and the times repeated to find the absolute quantity—or expressed in figures: \$4 × 3 = \$12. In the inverse operation (division) we have *two* of these things given, viz: \$4 and \$12, to find the third, viz: *three;* and both science and common sense demand that THIS *three* shall be found, and *not* a transformed three, as three dollars, or "three, four dollars," or three *anything else* in the whole realm of the *concrete.* Yet we have the astonishing statement that "the quotient is *not* an abstract number—it is three, four dollars." Expressed in symbols this would be, when the dividend is recalculated from its factors:—\$4 × 3 (\$4) = \$12—or in words, three dollars repeated three, four dollars times is equal to twelve dollars! The statement that "the divisor and dividend are *always* of the same name" will be referred to again.

(2) "The quotient is *always* equal to the dividend." "Get the children to see this, and when grown to men and women they will not make such mistakes"—as e.g., thinking that the quotient may be a pure number. That is: $\frac{a}{b}$ = quotient q (say): multiply equals by equals $\therefore a = bq$, but $q = a$ (the dividend). $\therefore a = ab$!, Or taking the professor's favourite example: $\frac{\$12}{\$4} = 3\,(\$4)$

$\therefore$ \$ 12 = 3 (\$4) × \$4 = unmiti-