of approach toward the outlet—is somewhat different from the case of frictional flow within a long cylindrical pipe, yet stability of equilibrium is regained in quite the same way; and fractional resistance is also present as a contributory factor, although of minor importance. It is virtually impossible to make the water flow from the wash-bowl in parallelism with the axis of the outlet. It insists on whirling helically to right or left, whichever way it happens to be started; and whichever way it whirls, it regains thereby its stability.

Within the helical whirl the pressure head, acting radially outward, is not everywhere equal. Owing to centrifugal

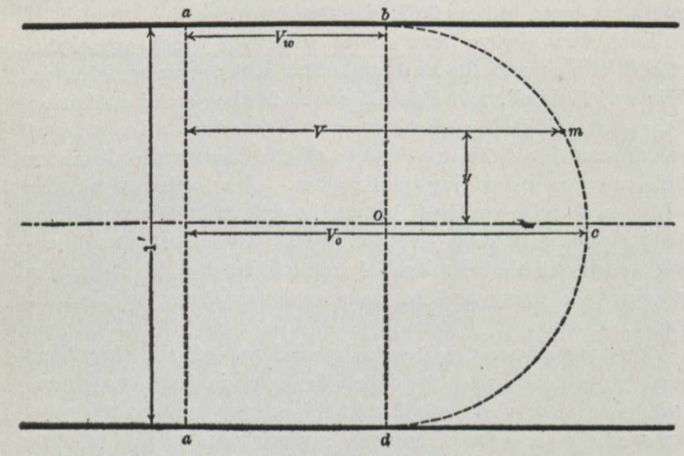

Fig. 1.

force, it is greater near the periphery. Equilibrium will be found when the rise in radial pressure, due to the greater radius and velocity of the whirl, as it passes from the axis to the wall of the pipe, just counterbalances the loss in velocity head due to the slower progress of the water along the pipe near the walls. The more the axial progress of the water is arrested by friction, the more it whirls, instead of progressing.

The energy of the whirl, however, cannot be all of that disappearing in the form of axial velocity. It must be something less than this, for the frictional resistance must be continually made good. The loss of axial velocity must go to two destinations, namely, the whirl, and the wall friction. The current parallel with the axis is both diverted and retarded by the wall friction. In other words, the oblique velocity along the helical path cannot be any greater, and must be somewhat less, than the maximum, axially-directed velocity in the center. Yet this oblique velocity is itself made up of two components, namely, (a) the velocity at any point in a direction parallel with the axis (that reported by the pitometer observations), and (b) the transverse velocity of whirl. That is, the helical velocity at any point in the pipe outside of the axis must be less than the velocity in the axis; and, because it is helical, instead of parallel with the axis, its axial component must be less, again, than its complete oblique self.

It is this second discount from the original maximum—the measurable axial velocity—which alone the pitometer measures in the traverse of the pipe away from the axis; but it is the first discount from the maximum velocity only, down to the oblique, helical velocity (only one component of which the pitometer measures), which supplies the energy visible in the whirl.

A mathematical solution of the situation will probably make it clearer. Let Fig. 1 represent the longitudinal section of a cylindrical conduit, having a diameter of 1 ft. Let Vc be the axial velocity therein, represented by the distance from the line, aa, to the point, c. Let Vw be the velocity, parallel to the axis, of the filaments touching the walls, represence by the distance from the line, aa, to the line, bd. For the present it is sufficient to assume, as determined by observation, that $Vw = \frac{1}{2} Vc.$

Let V be the velocity parallel with the axis at any point, m, in the cross-section of the pipe, at a distance, y, from the axis. Then at every such point the true velocity of the water will be a helical one, U, greater than V, because including some portion of the tangential component, yet less than Vc, because constantly contributing energy toward the wall. It is the deficit in velocity head of U below Vc which must be made up in centrifugal force, in order to counterbalance the excessive pressure head near the wall; yet this centrifugal force is the result of a tangential velocity which is itself a component of U.

Let v be the tangential velocity at any point. Then

$$U^2 = V^2 + v^2 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Therefore, the deficit in linear velocity head which creates the deficit in pressure head necessarily counterbalanced by centrifugal force, is

$$\frac{1}{2g}(Vc_2 - U^2) = \frac{1}{2g}(Vc_2 - V^2 - v^2) \ldots\ldots (2)$$

The centrifugal force which must be developed to counterbalance this, if stated in the form of a differential increase as one passes from the axis outwardly, is

$$dPc = \frac{v^2}{y} dM = \frac{v^2}{y}\frac{dW}{g} = 2\frac{v^2}{2g}\,0.433\,\frac{dy}{y} \ldots\ldots (3)$$

wherein 0.433 is the weight of a prism of water 1 sq. in. in cross-section and 1 ft. long.

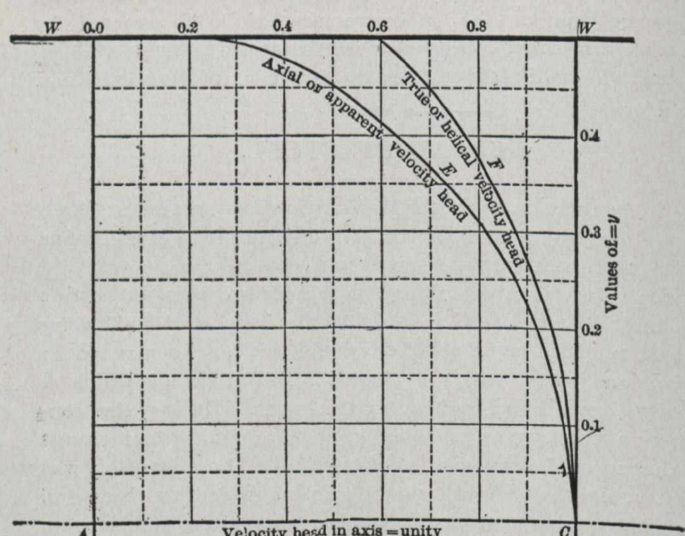

Fig. 2.

According to the paper already referred to, the curve, bcd, of Fig. 1 is normally an ellipse. If the horizontal scale for velocities be chosen so that Vc is represented by a length equal to the pipe diameter, the problem will be simplified further by making the curve, bcd, a semicircle. As the pipe diameter has already been assumed to be unity, this is tantamount to assuming that the axial velocity, $Vc = \sqrt{2g} = 8.02$ ft. per sec., and that all other axial velocities which might occur in practice would beget phenomena everywhere proportional thereto—an assumption which seems to be justified by the facts now available. In that case,