

§6. In (11) and (16) we have two equations, with the two unknown quantities y and t . The equations may be written

$$\begin{cases} 8kyt^2 + myt = n \\ vyt^2 + qyt = r \end{cases} \quad (17)$$

and

where

$$m = 50y - \frac{2}{5}p_1,$$

$$n = 8k^3 + p_5y,$$

$$v = y\left(50y - \frac{2}{5}p_1\right)^2 - 256k^4$$

$$= 2500y^3 - 40p_1y^2 + \frac{4}{25}p_1^2y - 256k^4,$$

$$q = -2(16k^3 + p_5y)\left(50y - \frac{2}{5}p_1\right) - 256k^3y$$

$$= -100p_5y^2 + \left(\frac{4}{5}p_1p_5 - 1856k^3\right)y + \frac{64}{5}k^3p_1,$$

$$r = 100k^3y^3 - \left(\frac{24p_1k^3}{5} + p_5^2\right)y^2 + \left(\frac{4}{25}p_1^2k^3 - 32k^3p_5\right)y - 256k^4.$$

§7. The elimination of t from the equations (17) gives us

$$(vn - 8kr)^2 + y(qn - mr)(vm - 8kq) = 0.$$

From the values of m , n , v , q and r in §6,

$$vn = 2500p_5y^4 + (20000k^3 - 40p_1p_5)y^3 + \left(\frac{4}{25}p_1^2p_5 - 320p_1k^3\right)y^2 \quad (18)$$

$$+ \left(\frac{32}{25}p_1^2k^3 - 256k^4p_5\right)y - 2048k^4;$$

$$8kr = 800k^3y^3 - \left(\frac{192}{5}p_1k^3 + 8p_5^2k\right)y^2 + \left(\frac{32}{25}p_1^2k^3 - 256k^4p_5\right)y - 2048k^4;$$

$$qn = -100p_5^2y^3 + \left(\frac{4}{5}p_1p_5^2 - 2656k^3p_5\right)y^2 + \left(\frac{96}{5}p_5k^3p_1 - 14848k^3\right)y + 512k^4p_1;$$

$$mr = 5000k^3y^4 - (280p_1k^3 + 50p_5^2)y^3 + \left(\frac{248}{25}p_1^2k^3 - 1600k^3p_5 + \frac{2}{5}p_1p_5^2\right)y^2$$

$$- \left(12800k^4 + \frac{8}{125}p_1^3k^3 - \frac{64}{5}k^3p_1p_5\right)y + \frac{512}{5}k^4p_1;$$

$$vm = 125000y^4 - 3000p_1y^3 + 24p_1^2y^2 - \left(12800k^4 + \frac{8}{125}p_1^3\right)y + \frac{512}{5}p_1k^4;$$

$$8kq = -800p_5ky^3 + \left(\frac{32}{5}kp_1p_5 - 14848k^4\right)y + \frac{512}{5}p_1k^4.$$

$$\therefore vn - 8kr = y^2 \left\{ 2500p_5y^2 + (19200k^3 - 40p_1p_5)y + \left(\frac{4}{25}p_1^2p_5 - \frac{1048}{5}p_1k^3 + 8p_5^2k\right) \right\},$$