

or as in such an obtuse angle as  $9^{\circ} 32'$  the sine and chord commence to differ appreciably—we may take the more exact value by (1) :

$$F = \frac{2x}{\sin a} = \frac{9.4}{\sin 9^{\circ} 32'} = 56.8$$

T is in this case 10 inches or .83 ft.

Substituting in (3)  $s_1^2 = F \sqrt{\frac{T}{\mu}} = 23.8$

$$\frac{F D}{100} = a, \text{ therefore } D = \frac{100 a}{F} \pm (8) \\ = 16^{\circ}.78 \text{ or } 16^{\circ} 44'$$

We have still to obtain the position of the 1 in 10 frog—between the two turnout rails. Let  $x$  be the unknown distance from the point of the last one considered (the 1 in 6), and  $D$  the curvature per 100 ft. over this piece  $x$ .

Then  $\frac{x D}{100}$  will be the deflection of the South turnout between the two frogs, and the total deflection from main line to point of 1 in 10 frog will be  $\frac{x D}{100} + 9^{\circ} 32'$ . On the north turnout the deflection to same point will be that for a distance  $27 + 33 + x = x + 60$ , and will be represented by the expression  $\frac{x + 60}{100} \times 6^{\circ}.12$ . But by hypothesis the difference between these is the angle of the frog or  $5^{\circ} 44'$  hence the equation :

$$\frac{x D}{100} + 9.53 - 6.12 = \frac{x + 60}{100} \times 6.12 \\ x D + 953 - 6.12 x - 367 = 573 \\ x D = 6.12 x - 13 \\ D = \frac{6.12 x - 13}{x}$$

Again we have for the lateral position of the frog from south rail of main line :—

$$\text{on the north turnout } \left( \frac{x + 60}{100} \right)^2 \times 5.3$$

$$\text{on the south } " \quad x \sin 9^{\circ} 32' + 87 D \left( \frac{x}{100} \right)^2$$

$$\text{equating } \frac{(60 + x)^2 \times 5.3}{10000} = .1656 x + \frac{.87 D x^2}{10000} \\ (60 + x)^2 \times 5.3 = 1656 x + .87 D x^2 \\ 19080 + 636 x + 5.3 x^2 = 1656 x + .87 D x^2$$

substituting the value of  $D$  in terms of  $x$  above,

$$5.3 x^2 + 636 x + 19080 = .87 x^2 \times \frac{6.1 x - 13}{x} + 1656 x \\ = 5.3 x^2 - 11.3 x + 1656 x \\ 1009 x = 19080 \\ x = 18.9,$$

Substituting in the equation of  $D$  in terms of  $x$ ,

$$D = \frac{6.1 x - 13}{x} = \frac{115.3 - 13}{18.9} = 5^{\circ}.41 \text{ or } 5^{\circ} 25'$$

Substituting in the expression for the offset or lateral position of the frog point, we have :

$$0 = \left( \frac{60 + x}{100} \right)^2 5.3 = .789^2 \times 5.3 = 3.3 \text{ ft.}$$

Fig. 7 is a combination of the arrangements in Figs. 5 and 6, and is an example of a 4 throw switch, something not often seen in actual practice. For reasons explained further on, it would not be advisable to break up the sharp curve forming the lead of south turnout by inserting another frog. A 1 in 8 has therefore been instituted.

Making  $F = 75.2$ ,  $D = 9^{\circ} 30'$ ,  $S = 31.6$ ,  $L = 43.6$

The only element left not solved as in the preceding is the position of frog between the extreme north and south turnouts and its angle.