

par les seules données A, B, c , sans autre angle ou ligne quelconque, mais la ligne c est heterogenee avec les nombres A, B, C ; et si on avoit une equation quelconque entre A, B, C et c , on en pourroit tirer la valeur de c en A, B, C , d'où il resulteroit que c est egale à un nombre, ce qui est absurde. Donc c ne peut entrer dans la valeur de C , et on a simplement $C = \phi(A, B)$." Leslie committed the unaccountable mistake of supposing the argument here stated, to be, that "that the line c is of a nature heterogeneous to the angles A and B , and therefore cannot be compounded with these quantities"—whereas the argument plainly is, that c , which is a line, cannot be expressed in terms solely of A, B , and C , which are numbers. "The quantities A, B, C ," says Playfair, in his exposition of Legendre's reasoning, "are angles; they are of the same nature with numbers, or mere expressions of ratio, and, according to the language of Algebra, are of no dimension. The quantity c , on the other hand, is the base of a triangle, that is to say, a straight line, or a quantity of one dimension. Of the four quantities, therefore, A, B, C, c , the first three are of no dimensions, and the fourth or last is of one dimension. No equation therefore can exist, involving all these four quantities and them only: for if there did, a value of c might be found in terms of A, B , and C ; and c would therefore be equal to a quantity of no dimensions: which is impossible."

In this reasoning it is assumed, that, because C is *determined* by A, B, c , therefore C can be *expressed* in terms of A, B, c . Now Legendre does not prove that when a quantity is determined by certain others, it can be expressed in terms of them; and I affirm that *such a principle, without limitation, is not true*.

For example, consider the angle C of the triangle ABC . And let it be observed that I mean the angle itself, that is, the inclination of a and b to one another, and not the numerical value of the angle, calculated upon the supposition that a right angle, or any other angle, has been assumed as a unit of measure. The angle C is *determined* by the sides a, b, c ; yet it cannot be expressed in terms of these quantities alone; because *the value of an angle can only be indicated by pointing out its relation to some other angle or angles*; and therefore cannot be expressed by means simply of lines. It is true that *the numerical value* of C may be expressed in terms of a, b , and c : viz, in an equation where only the ratios of a, b , and c occur, the ratios being numbers. Thus, if $b = \beta a$, and $c = \gamma a$, we might have

$$\text{numerical value of } C = f(\beta, \gamma)$$

But this is altogether a different thing from saying that C *itself*, the angle properly so called, the inclination of a and b to one another,