

$r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}}$ (45); multiplying (42) by r and substituting it in (40.)

$$S = \frac{rl - a}{r - 1} \quad (46); \quad a = rl - S(r - 1) \quad (47); \quad l = S - \frac{S - a}{r}$$

$$(48); \quad r = \frac{S - a}{S - l} \quad (49). \quad \text{When } n \text{ is infinite, and } r \text{ a proper frac-}$$

tion, ar^n in (40) becomes $= 0$. Hence for an *Infinite Series*

$$S = \frac{a}{1 - r} \quad (50); \quad a = S(1 - r) \quad (51); \quad \text{and } r = \frac{S - a}{S} \quad (52).$$

VII. ANNUITIES AT SIMPLE INTEREST.

Let A = a single payment of the Annuity, M = Amount, t = number of payments, and r = Interest of *one* pound for *one* period. Then when the annuity is forborne any number of payments, the last payment being received at the time it falls due, $= A$; last but one $= A + Ar$, last but two $= A + 2Ar$, last but three $= A + 3Ar$, 1st $= A + (t - 1)Ar$; hence $M = A + (A + Ar) + (A + 2Ar) + (A + 3Ar) + \dots (A + (t - 1)Ar)$. Whence from (28), $M = At \left(1 + \frac{(t - 1)r}{2}\right)$ (53),

$$A = \frac{2M}{t(2 + r(t - 1))} \quad (54); \quad r = \frac{2(M - At)}{At(t - 1)} \quad (55); \quad \text{and } t =$$

$$\frac{\sqrt{8r \frac{M}{A} + (2 - r)^2} - (2 - r)}{2r} \quad (56); \quad \text{Let } v = \text{present value of}$$

$$\text{an annuity to continue any number of payments; from (6) } v(1 + rt) = M = At \left(1 + \frac{(t - 1)r}{2}\right); \quad \text{hence } v = \frac{2 + (t - 1)r}{2}$$

$$\left(\frac{At}{1 + rt}\right) \quad (57); \quad r = \frac{2(At - v)}{(2v - (t - 1)A)t} \quad (58); \quad A = \frac{2v(tr + 1)}{(2 + (t - 1)r)} \quad (59).$$