I believe the difficulty has arisen from the lack of a proper root symbol, but of this hereafter; to discuss it would lead me beyond the limits of Algebra and afar from the purpose of the present paper. What this lack has done in mathematics, may be judged from its leading such an author as Mr. Todhunter into the inconsistency of accepting 49 as a root of $x + 4\sqrt{x} = 21$, and rejecting 18 as a root of $x + \sqrt{(5x + 10)} = 8$. (Algebra, pp. 169 and 170.)

In $x + \sqrt{(x^2 - a^2)} = b$, put $y = \sqrt{(x^2 - a^2)}$, $\therefore x + y = b$ and $x^2 - y^2 = a^2$ or $x - y = \frac{a^2}{b}$. If $a^2 > b^2$, y is negative, but what writer on Algebra would reject such negative value as the solution of $x + y = b$, $x - y = \frac{a^2}{b}$, and hold that $x - y = b$, $x + y = \frac{a^2}{b}$ had been solved. Yet this is what they virtually do when these equations are written in the form involving the radical sign; they wholly disregard the principle of the equivalence of equations. To take a particular case let $x + \sqrt{(x^2 - 24)} = 4$ be proposed. According to the common algebras, there is no solution, 5 being the root of $x - \sqrt{(x^2 - 24)} = 4$. Instead propose $x + y = 4$, $x^2 - y^2 = 24$. No hesitation, $x = 5$ and $y = -1$. *The equations are algebraically identical.* Next let $z + \sqrt{(z^2 + 24)} = 4$, be proposed. Again there will be no hesitation, $z = -1$. In this equation put $z = \sqrt{(x^2 - 24)}$, and there will appear the original equation in x, thus showing that the equations are equivalent. $x - \sqrt{(x^2 - 24)} = 4$ would need $z - \sqrt{(z^2 + 24)} = 4$ and by the common view this actually cannot be solved. If there were proper root and 'affection' symbols, it would at once appear that the equation solved and the substitutions made were $x + (-\sqrt{}) (x^2 - 24) = 4$, $y = (-\sqrt{}) (x^2 - 24)$, and $z = (-\sqrt{}) (x^2 - 24)$.

For the sake of clearness in the reasoning, I have confined myself, in the foregoing strictures, to the form, $x + \sqrt{(x^2 - a^2)} = b$, but the principle for which I contend, is the same for all equations of the type $F(x) + \sqrt{\{f(x)\}} = c$: it is merely that in $+\sqrt{}$, unless otherwise stated, or required from the value of the problem, either root may taken, $+$ being the symbol of addition of that root, but not of its 'affection.'

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2. *To the Editor of the Journal of Education:*

SIR,—As Mr. Glashan's method of solving the equations $x^2 + y = 11$, $x + y^2 = 7$ as given in your last number may be a little too abstruse for some of the readers of the *Journal*, I send the following. One of the roots is found by the method of approximation and the others by factoring. The following is the rule for finding a root of an equation by approximation—"Find by trial two numbers as near the true roots as possible, and substitute them in the given equation instead of the unknown quantity. Then as the difference of these results is to the difference of the two assumed numbers, so is the difference between the true result and either of the former, to the correction of the number belonging to the result used. If the number thus found and the nearest of the two former, or any other more accurate, be taken as the assumed roots, and the operation be repeated a value of the unknown will be obtained still more correct than the first, and so on.

$$x^2 + y = 11 \quad \therefore y = 11 - x^2.$$
$$x + y^2 = 7 \qquad y^2 = 7 - x.$$
$$\therefore x^4 - 22x^2 + x + 114 = 0.$$
$$\text{or } (x - 3)(x^3 + 3x^2 - 13x - 38) = 0.$$

This is satisfied by $x = 3$.

$$x^3 + 3x^2 - 13x = 38.$$

To solve the latter equation, it is found by trial that the value of x lies between 3 and 4. Take these as the assumed numbers.

When $x = 3$, $x^3 + 3x^2 - 13x = 15$.
When $x = 4$, $x^3 + 3x^2 - 13x = 60$.

$$\text{Then} \left\{ \begin{array}{ccccc} 60 & & 4 & & 38 \\ 15 & & 3 & & 15 \\ \hline 45 & : & 1 & :: & 23 : \cdot 5+. \end{array} \right.$$

Hence $x = 3\cdot5+$.

Again take 3·5, 3·6 as the assumed numbers and we shall find, $x = 3\cdot584+$, and so on to any number of decimal places.

Dividing $x^2 + 13x^2 - 13x - 38$ by $x - 3\cdot584$ we obtain

$$x^2 + 6\cdot584x + 10\cdot6013.$$

Solving as a common quadratic

$$x^2 + 6\cdot584x + 10.6013 = 0.$$
$$x = -3\cdot779 \text{ or } -2\cdot805.$$

Hence the four roots are determined.

The following geometrical theorems are proposed to the mathematical readers of the *Journal*:—

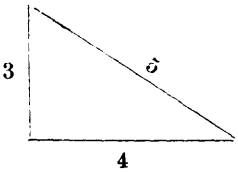
From a point A without a circle (centre O) draw the tangent AC and the line ABO, cutting the circumference in B. Bisect AC in D. Let fall the perpendiculars CEDF on AO. Draw FG touching the circle in G. Join GE and produce to meet the circle in H. Then

(1.) HB is a side of the inscribed square.
(1.) $HB^2 = HE, HG$,
If $EG = OE$ then
(3.) HG is a side of the inscribed equal triangle.
(4.) $GB^2 = GH, GE - GB, BH$.

W.

August 12.

SOLUTION TO THE BELFAST COLLEGE PROBLEM.



Let x be the quantity to be added to the perpendicular. Then $(3 + x)^2 + 4^2 = \square$, or $x^2 + 6x + 25 = \square$; let $x - z =$ the side of that square and we have $x^2 + 6x + 25 = (x - z)^2$, or $x^2 + 6x + 25 = x^2 - 2xz + z^2$, or $6x + 2xz = z^2 - 25 \therefore x = \frac{z^2 - 25}{6 + 2x}$, (I.)

Again let twice x be added to the perpendicular, and we have $(3 + 2x)^2 + 4^2 = \square$, or $4x^2 + 12x + 25 = \square$, substituting the value of x, we have $4\left(\frac{z^2 - 25}{6 + 2z}\right)^2 + 12\left(\frac{z^2 - 25}{6 + 2z}\right) + 25 = \square$; expanding and dividing by 4 we have $\frac{z^4 + 6z^3 - 7z^2 + 400}{36 + 24z + 4z^2} = \square$, and dividing by $36 + 24z + 4z^2$, which is a square, we have $z^4 + 6z^3 - 7z^2 + 400 = \square$; let the side of this square be $20 + z^2$ and we get $z^4 + 6z^3 - 7z^2 + 400 = (20 + z^2)^2$, $\therefore$ or $6z^3 = 47z^2$, or $z = \frac{47}{6}$, and substituting this value of z for z in (I), we have $x = \frac{1309}{780}$. *Ans.*

W. G. KIDD.

Fergus, August 13, 1870.

III. Miscellaneous.

1. THE CHILDREN'S PRAYER.

They were all alone in the parlour,
Mary, and Alice, and Will;
But a shadow clouded their faces,
And for once their tongues were still.

Till Willie sobbed, "Mamma said baby might die,
Our beautiful Bell;
But I'm sure God will not take her,
He knows we love her so well.

"And yet if He wants her in heaven,
'Tis better for her to go,
And live with the Saviour for ever,
He loves little children so.

"Perheps if we ask Him to spare her,
He will listen while we pray,
For mamma says that He always hears
All the prayers that children say."

So with tear-dimmed eyes and folded hands,
Together they knelt and prayed,
And God looked down in mercy, and heard
The simple words they said:

"O God please let the baby live,
If it be Thy holy will;
But if Thou takes't her up to heaven,
Help us to bless Thee still."

And they slept that night without fear of harm,
For they trusted in God above,
And knew whatever He sent to them
Was sent in mercy and love.

And when morning came, in their mother's face
They read their answer well,
And thanked the dear Lord who heard their prayer
And spared the baby Bell.

'Tis years since then, but they ne'er forget
The lesson they learned that night,
The prayers of God's children, howevor weak,
Are precious in his sight.